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COMPARISON OF WIRE—PLATE AND PLATE—PLATE ELECTROSTATIC PRECIPITATORS IN TURBULENT FLOW

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Summary

The transport of pre-charged particles in a wire—plate precipitator in turbulent flow is investigated by numerical solution of the convective diffusion equation. For ease of computation the high voltage wire electrodes are replaced by strip electrodes giving essentially the same potential distribution. It is shown that for equivalent values of the Deutsch and Peclet numbers, the efficiency of the wire—plate system agrees closely with that of a plate—plate system, provided the comparison is made on the basis of equal values of the space-averaged field at the collector. The study does not include corona charging as normally encountered.

Introduction

For the collection process in electrostatic precipitators the long established Deutsch model [1], which is still widely used for sizing and assessing the performance of precipitators, makes the crude assumption that there is complete mixing transverse to the flow direction (but no axial diffusion is considered). As a result of this assumption the particle concentration profile is flat at every axial station and the concentration, n, decays exponentially in the flow direction according to the well known Deutsch formula $n=n_0 \exp -(w_E\, x/ud)$. Here n_0 denotes the initial particle concentration, w_E is the electric migration velocity of the particles (assumed monodisperse), u is the gas velocity (assumed uniform), d is the wire-to-plate spacing, and x denotes the streamwise coordinate.

However, to understand and model the collection process, and to improve on the crude assumption of the Deutsch model, a more detailed study of the physics of charged particle transport is necessary, including the effect of the nonuniform electric field and the turbulent gas flowfield of typical wire—duct precipitators.

In earlier work [2] an analysis was given of the case of pre-charged particles entering a parallel plate collector, in which the effect of turbulence was modeled in terms of a uniform turbulent diffusivity D. It was shown

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that the collection efficiency was a function not only of the Deutsch number $De \equiv (w_E\ x/ud)$, the nondimensional length, but also of the Peclet number $Pe \equiv (w_E d/D)$. In the laminar limit $Pe \to \infty$, the results show that 100% collection efficiency is obtained in a finite length De=1. In the opposite limit $Pe \to 0$ (but neglecting axial diffusion) the Deutsch efficiency formula $\eta=(1-\exp-De)$ is recovered. On this basis it was suggested that efficiencies significantly greater than predicted by the Deutsch model might be obtained by maintaining the turbulence intensity (and hence D) at low values. In recent work, Kihm et al. [3] studied the effect of taking account of the nonuniform electric field in a wire—plate collector with entering pre-charged particles for laminar flow (D=0). It was shown that when the effective migration velocity w_E was based on the space-average field E_{av} at the collector, then the efficiencies as a function of De collapsed onto the laminar limit of the parallel plate case for all wire—plate geometries examined.

In the present paper we examine the effect of taking account of both the nonuniform electric field and the turbulent gas flowfield assuming a uniform eddy diffusivity in a wire—plate collector with entering pre-charged particles. The convective diffusion equation considered here includes the axial diffusion term which is believed to be necessary for studying properly the particle transport in the wire—plate collector. This term was shown to be generally negligible in case of the plate—plate collector system [4]. The resulting elliptic equation is solved numerically by a computer code which can accomodate the more general problem of taking account of a space dependent diffusion coefficient.

Formulation

For a particle suspended in a turbulent gas flow, the total particle velocity is

$$\underset{\sim}{w}(\underset{\sim}{r},t) = \underset{\sim}{u}(\underset{\sim}{r},t) + \underset{\sim}{w}_E(\underset{\sim}{r}) \tag{1}$$

where $\underset{\sim}{u}$ is the gas velocity and $\underset{\sim}{w}_E$ is the migration velocity relative to the gas and is given, neglecting inertia and gravity, by equating the Coulomb force to the Stokes' drag:

$$\underset{\sim}{w}_E(\underset{\sim}{r}) = \frac{q\,\underset{\sim}{E}(\underset{\sim}{r})}{6\pi a_p \eta_g} \equiv \mu\,\underset{\sim}{E}(\underset{\sim}{r}) \tag{2}$$

Here q and a_p are the practicle charge and radius respectively, η_g is the gas velocity and μ is the particle mobility. The electric field $\underset{\sim}{E}$ and hence $\underset{\sim}{w}_E$ are assumed time independent.

For particles all having the same mobility, the particle concentration $n(\underset{\sim}{r},t)$ satisfies the continuity equation

$$\frac{\partial n}{\partial t} + \nabla \cdot n\underset{\sim}{w} = 0 \tag{3}$$

or, substituting from (1) and (2)

$$\frac{\partial n}{\partial t} + \nabla \cdot n\underset{\sim}{u} + \mu \nabla \cdot n\underset{\sim}{E} = 0 \tag{4}$$

Putting $n = \bar{n} + n'$, and $\bar{\underset{\sim}{u}} + \underset{\sim}{u}'$, where the overbar indicates the time-average, and the prime indicates the fluctuation about the mean, and taking the time-average, eqn. (4) yields

$$\nabla \cdot \bar{n}\, \bar{\underset{\sim}{u}} + \mu \nabla \cdot \bar{n}\, \underset{\sim}{\bar{E}} + \nabla \cdot \overline{n'\underset{\sim}{u}'} = 0 \tag{5}$$

For incompressible flow $\nabla \cdot \bar{\underset{\sim}{u}} = 0$ and neglecting particle space charge, which is justified for a dilute flow, $\nabla \cdot \underset{\sim}{E} = 0$.

Also, modeling the turbulent flux term in terms of an eddy diffusivity D

$$\overline{n'u'} = -D\, \nabla \bar{n} \tag{6}$$

then eqn. (5) reduces to the convective diffusion equation in the form

$$(\bar{\underset{\sim}{u}} + \mu \underset{\sim}{E}) \cdot \nabla \bar{n} - \nabla \cdot D\, \nabla \bar{n} = 0 \tag{7}$$

The geometry of the problem considered here is shown in Fig. 1. The high voltage electrodes, of the same polarity as the particle charge, are disposed along the flow axis (x) with spacing $2h$, the first electrode being located at the origin (x=0, y=0). The grounded collector plates lie along the planes $y = \pm d$. The mean gas velocity $\bar{u}$ is directed along the x-direction and is assumed uniform; thus boundary layers and the wakes of the high voltage electrodes are neglected. The eddy diffusivity is also assumed uniform and all quantities are assumed independent of the z-coordinate.

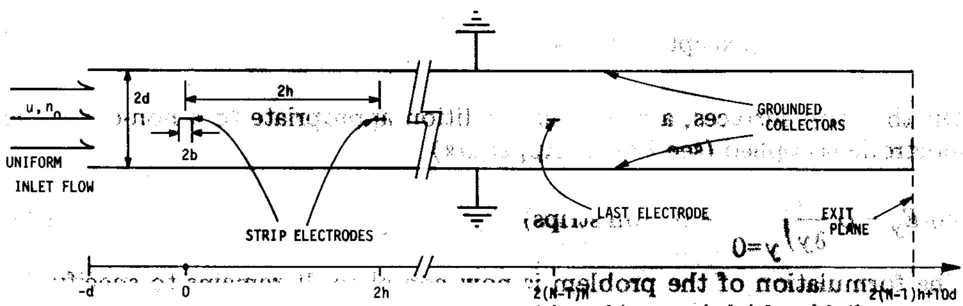


Fig. 1. Schematic of strip electrode configuration.

Then eqn. (7) becomes (omitting the over bars)

$$[u + \mu E_x(x,y)] \frac{\partial n}{\partial x} + \mu E_y(x,y) \frac{\partial n}{\partial y} - D\left(\frac{\partial^2 n}{\partial x^2} + \frac{\partial^2 n}{\partial y^2}\right) = 0 \tag{8}$$

Because, eqn. (8) is of elliptic type, boundary conditions on n must be specified at all the system boundaries. This equation type is consistent with the physical notion that the electrodes must have an upstream influence on the particle trajectories, and therefore on the particle concentration.

At the entrance, taken at $x = -d$, the inlet concentration n_0 is assumed uniform, i.e.

$$n\,(x = -d, y) = n_0 \tag{9a}$$

With this condition, and because of the symmetry of the problem, we have $n(x,y) = n(x,-y)$ so that it is sufficient to consider the solution of eqn. (8) only in the upper half plane $0 \leqslant y \leqslant d$.

A finite number N of high voltage electrodes is considered, and the exit plane (x=L) is taken at a position $10d$ beyong the last electrode position x=$2h$ (N–1). With this electrodeless exit section, the electric field at the exit is negligible so that the concentration profile should have relaxed to a uniform one. Thus the exit boundary condition is

$$\left.\frac{\partial n}{\partial y}\right|_{x=L} = 0 \tag{9b}$$

At the collecting electrode (y=d), the following boundary condition is imposed (see Ref. 2, Appendix)

$$\left.\frac{\partial n}{\partial y}\right|_{y=d} = 0 \tag{9c}$$

For ease of computation, the high voltage electrodes are taken as strips of width $2b$ in the x-direction and of infinitesimal thickness in the y-direction. Between the strip electrodes, symmetry requires the boundary condition

$$\left.\frac{\partial n}{\partial y}\right|_{y=0} = 0 \quad \text{(except on strips)} \tag{9d}$$

On the strip surfaces, a zero flux condition appropriate to a non-collecting electrode is applied (see Ref. 2, Appendix)

$$\left(n\mu E_y - D\,\frac{\partial n}{\partial y}\right)_{y=0} = 0 \quad \text{(on strips)} \tag{9e}$$

The formulation of the problem is now complete. It remains to specify the electric field, which is considered in the next section.

Electric field calculation

Commonly, in duct type precipitators the high voltage electrodes consist of cylindrical wires. Neglecting space charge the potential distribution is given by the solution of Laplace's equation $\nabla^2 V$=0.

For an infinite array of wires of radius a, located at x=0, $x = \pm\,2mh$, y=0 between ground planes at $y = \pm\,d$, the potential distribution is given for $(a/d) << 1$ by the use of complex mapping and superposition [5] as

$$V(x,y) = V_0 \frac{\sum_{m=-\infty}^{\infty} \ln \left\{ \dfrac{\cosh\,[\pi(x-2mh)/2d] + \cos\,(\pi y/d)}{\cosh\,[\pi(x-2mh)/2d] - \cos\,(\pi y/2d)} \right\}}{\sum_{m=-\infty}^{\infty} \ln \left\{ \dfrac{\cosh\,(\pi mh/d) + \cos\,(\pi a/2d)}{\cosh\,(\pi mh/d) - \cos\,(\pi a/2d)} \right\}} \tag{10}$$

where V_0 is the potential of the wires. This expression was used in the previous treatment [3] of the wire– plate precipitator in laminar flow.

In the present study it was found convenient to replace the cylindrical electrodes by strip electrodes of width $2b$ and infinitesimal thickness in order to avoid the difficulties associated with treating a circular boundary with a rectangular computatrional grid.

Accordingly, the Laplace equation

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0 \tag{11}$$

was solved numerically for the geometry shown in Fig. 1. A total of N strip electrodes of width $2b$ and separation $2h$ are located at (x=0, $2mh$, y=0) and the ground electrodes lie in the planes $y = \pm d$. The ground electrodes are extended in both directions by $10d$ beyond the end electrodes, so that a zero potential boundary condition can be imposed at these positions without significantly affecting the solution in the region of interest. Also, because of the symmetry about the plane y=0, it is sufficient to solve only for the half-plane $0 \leqslant y \leqslant d$.

Then the boundary conditions for eqn. (11) become

$$V(x = -10d, y) = 0 \tag{12a}$$

$$V(x = 2h\,(N-1) + 10d, y) = 0 \tag{12b}$$

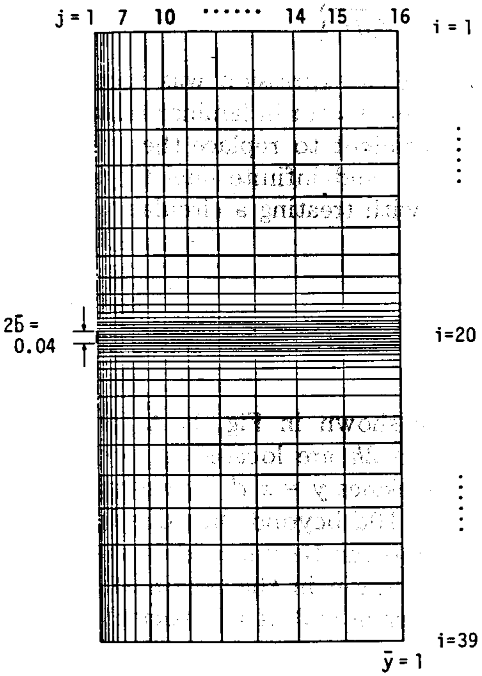
$$V(x, y = d) = 0 \tag{12c}$$

$$\left.\frac{\partial V}{\partial y}\right|_{y=0} = 0 \quad \text{(except on strips)} \tag{12d}$$

$$V(x, y=0) = V_0 \quad \text{(on strips)} \tag{12e}$$

Numerical solutions of eqn. (11) subject to eqns. (12) were computed using the method of successive over-relaxation with a two-dimensional graded grid [6]. The mesh size was decreased towards the electrode where the potential gradients are large, as shown in Fig. 2.

Since it was desired to make the potential distribution for the strips match, as closely as possible, the distribution given by wires, a number of cases were run for different values of strip width (b/d) for comparison with the potential distribution given by typical values of wire diameter



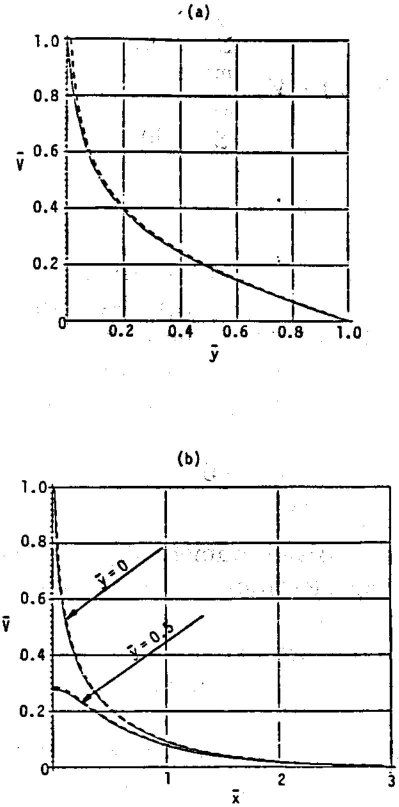


Fig. 2. Mesh structure around each electrode used to calculate both the potential field and the particle concentration distribution.

Fig. 3. Comparison of potentials for wire ($\bar{a}$ = 0.0133, broken line) and strip ($\bar{b}$ = 0.02, solid line) between grounded planes. (a) $\bar{V}$ ($\bar{x}$ = 0, $\bar{y}$); (b) $\bar{V}$ ($\bar{x}$,$\bar{y}$ = 0) upper curves; $\bar{V}$ ($\bar{x}$,$\bar{y}$ = 1/2) lower curves.

(a/d) given by eqn. (10). Figure 3 shows a comparison for (h/d)=1 of the potential distributions as functions of $\bar{x} = (x/d)$ and $\bar{y} = (y/d)$ for the cases (a/d) = 0.0133 and (b/d) = 0.02. It is seen that with this choice of strip width, the potential distributions of the strip and the wire match very closely except very near to the electrodes. In general, for $(a/d) \ll 1$, it was found that the potentials matched closely when $(b/a) \approx 1.5$.

Having computed the potential, the electric field components are found by numerical differentiation for substitution in eqn. (8).

Solution method

The governing equations (8) and (9) are non-dimensionalized using the dimensionless variables $\bar{x} = (x/d)$, $\bar{y} = (y/d)$, $\bar{L} = (L/d)$, $\bar{n} = (n/n_0)$, and

$\bar{E} = (E/E_0)$ where $E_0 \equiv (V_0/d)$, as follows

$$\left[\frac{1}{\epsilon_0} + \bar{E}_x(\bar{x},\bar{y})\right] \frac{\partial \bar{n}}{\partial \bar{x}} + \bar{E}_y(\bar{x},\bar{y}) \frac{\partial \bar{n}}{\partial \bar{y}} - \frac{1}{Pe_0}\left[\frac{\partial^2 \bar{n}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{n}}{\partial \bar{y}^2}\right] = 0 \tag{13}$$

$$\bar{n}\ (\bar{x} = -1, \bar{y}) = 1 \tag{14a}$$

$$\left.\frac{\partial \bar{n}}{\partial \bar{y}}\right|_{\bar{x}=\bar{L}} = 0 \tag{14b}$$

$$\left.\frac{\partial \bar{n}}{\partial \bar{y}}\right|_{\bar{y}=1} = 0 \tag{14c}$$

$$\left.\frac{\partial \bar{n}}{\partial \bar{y}}\right|_{\bar{y}=0} = 0 \quad \text{(except on strips)} \tag{14d}$$

$$\left(Pe_0\, \bar{n} - \frac{\partial \bar{n}}{\partial \bar{y}}\right)_{\bar{y}=0} = 0 \quad \text{(on strips)} \tag{14e}$$

Here

$$\epsilon_0 \equiv (w_0/u) \tag{15}$$

where

$$w_0 = (\mu\ V_0/d) \tag{15a}$$

and

$$Pe_0 \equiv (w_0 d/D) \tag{16}$$

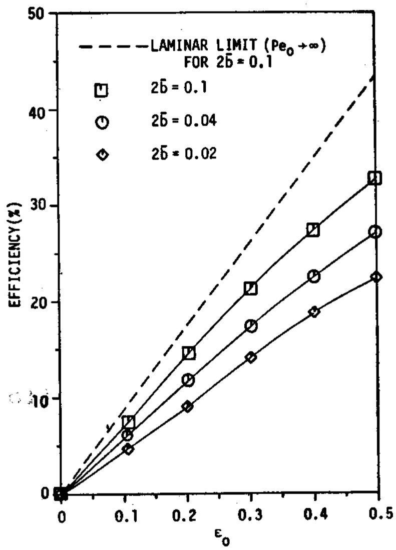
Thus, apart from geometric quantities, there are two parameters in the problem, ϵ_0 and Pe_0.

Equation (13) was solved by numerical iteration using successive over-relaxation (SOR) with the same graded grid as is used for the potential calculations. The total dimensions of the mesh are 130 $\times$ 16 for the single strip electrode system, and 830 $\times$ 16 for the multi-strip electrode system (N = 20 strips) discussed below. The grading ratios are 1.234 and 1.239 in the x and y directions respectively. The accuracy of computation is set by requiring that the maximum difference between the $\bar{n}$ values for successive iteration steps is less than 10^{-6} for all meshpoints. The optimal value of the SOR factor is chosen for most rapid convergence by inspecting the maximum differences between the first few iterations.

Results for single strip electrode

It is instructive to discuss the solution for a single strip electrode system before treating the multiple strip case. Figure 4 shows the collection efficiency as a function of ϵ_0 for the typical value Pe_0 = 2.5 and for three values of strip width $2\bar{b}$. As expected, the efficiency increases with strip width because, for the same applied potential, a wider strip produces a precipitating field which extends over a larger spatial extent.

The collection efficiency versus ϵ_0 for several values of Peclet number is shown in Fig. 5, for the case $2\bar{b} = 0.04$. It is seen that the efficiency increases and approaches the laminar limit as the gas flow becomes more laminar ($D \to 0, Pe_0 \to \infty$).



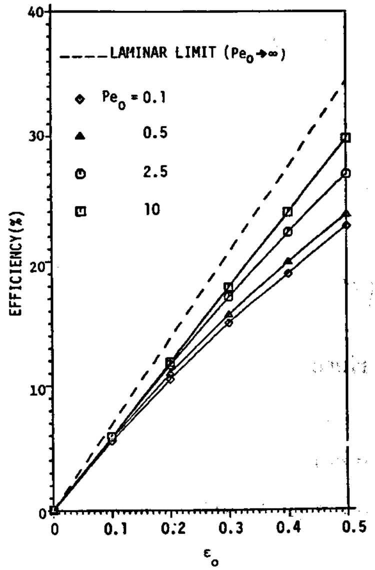


Fig. 4. Efficiency for single wire—plate collector with $Pe_0 = 2.5$ versus ϵ_0 as a function of wire size $\bar{a} = 0.667\ \bar{b}$.

Fig. 5. Efficiency for single wire—plate collector with finite diffusivity versus ϵ_0 compared with laminar wire—plate collector.

Results for multistrip electrodes

Computations have been made for the case of 20 strip electrodes with spacing equal to the duct width $(2h/d) = 1$, and of width $(2b/d) = 0.04$, corresponding to a wire diameter $(2a/d) \approx 0.027$.

Figure 6 shows the particle concentration as a function of $\bar{x}$ in four planes, $\bar{y} = 0$, 0.15, 0.5 and 1.0, for the typical parameter values $\epsilon_0 = 0.1$ and $Pe_0 = 2.5$. It is seen that, as expected, the particle concentration generally decreases along the flow direction $(\bar{x})$ for all $\bar{y}$ planes, and increases towards the collector $(\bar{y} = 1)$. However, the concentration profile in the x-direction shows marked periodic behavior near the center plane (small $\bar{y}$) while near the collector $(\bar{y} = 1)$ it decreases monotonically.

Along the centerline $(\bar{y} = 0)$ the concentration decreases suddenly near the upstream edge of the strip where there is a strong repulsive force on the particles. Downstream of an electrode the concentration increases again due to turbulent diffusion into the region of low field midway between electrodes, until the upstream edge of the next strip is approached. This is the origin of the successively decreasing peaks in $\bar{n}$ $(\bar{x})$.

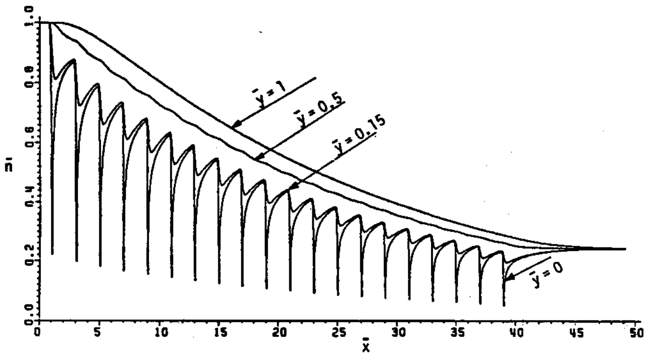


Fig. 6. Distribution of particle concentration as a function of $\bar{x}$ in the four planes $\bar{y}$ = 0, 0.15, 0.5, 1. [h/d=1, b/d=0.02, $\epsilon_0 = w_0/u_0 = 0.1$, $Pe_0 = 2.5$].

Far downstream of the last electrode the concentration tends to a constant value, independent of $\bar{y}$, since there is negligible electric field and turbulent diffusion causes the profile to be flat. Solutions obtained with the exit plane taken at positions either $5d$ or $20d$ beyond the last electrode show a negligible effect on the concentration profiles, thereby justifying the assumed downstream boundary condition (14b) for the elliptic problem.

For the same geometric parameters as in Fig. 6, Fig. 7 shows the efficiency η and penetration $(1-\eta)$ as a function of the Deutsch number $De_0 \equiv (w_0 x/ud)$, a dimensionless measure of precipitator length in the flow direction, for the typical value ϵ_0 = 0.1 and for several values of the Peclet number Pe_0. It should be noted that all three quantities De_0, ϵ_0 and Pe_0 are based on the migration velocity $w_0 = \mu E_0$ corresponding to the "applied field" $E_0 = (V_0/d)$. The knees in the curves at $De_0 \sim 4$, beyond which the efficiency becomes constant, corresponds to the x-coordinate of the last electrode. The general trends of the efficiency with De_0 and Pe_0 are similar to those of the parallel plate collector based on the same value of E_0, but for equal values of Pe_0, the efficiency is significantly lower. This is to be expected because, for equal applied fields the strip-electrode geometry has a significantly lower volume-averaged field than the uniform field E_0 of the plate—plate collector.

As was found for the laminar case with a nonuniform field [3], it is instructive to renormalize the results on the basis of the space-averaged field at the collector surface, $E_{av} = \langle E_y\ (x,y{=}d) \rangle$. For the geometry considered above, the ratio $(E_{av}/E_0) \approx 0.3$. The results are then given in terms of the re-scaled parameters De_{av}, ϵ_{av} and Pe_{av} based on $w_{av} \equiv \mu E_{av}$.

This is done in Fig. 8, which also shows the efficiency curves for the

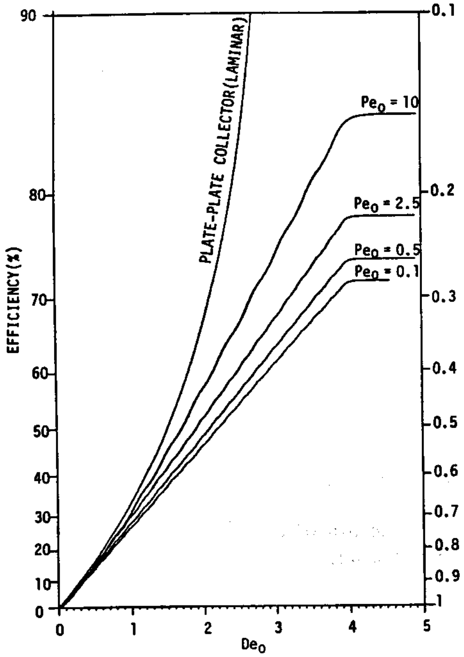


Fig. 7. Efficiency for wire—plate collector with finite diffusivity versus De_0 and several values of Pe_0. [$h/d = 1$, $b/d = 0.02$, $\epsilon_0 = w_0/u_0 = 0.1$].

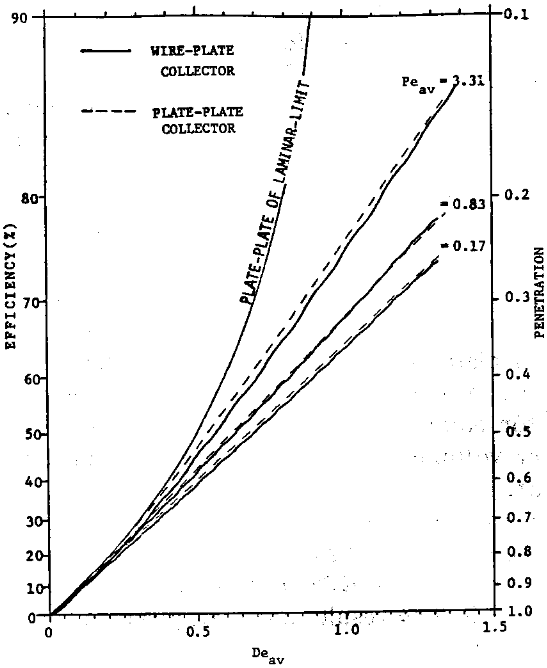


Fig. 8. Efficiency for wire—plate collector with finite diffusivity versus De_{av} compared with plate—plate collector. [h/d=1, b/d=0.02, $\epsilon = w_{av}/u_0 = 0.33$].

uniform field case, based on the uniform field E_0. The close agreement evident in Fig. 8, shows that for the purposes of calculating the efficiency in a wire—plate precipitator, it is a good approximation to use the simpler, uniform field theory [2] provided that the migration velocity is calculated from the space-averaged electric field at the collector. This, of course, is not an exact result, but appears to be a good approximation for the range of precipitator geometries usually encountered.

Summary and conclusion

The collection efficiency of a multi-strip—plate system as a collector for precharged monodisperse particles in turbulent flow, having uniform eddy diffusivity, has been investigated by numerical integration of the convective diffusion equation for the particle concentration.

It is found that, for calculation purposes, a wire—plate system can be replaced by a strip—plate system with the ratio of strip width to wire diameter $(2b/2a)=1.5$. Under this condition the potential distributions of the two geometries are closely matched, while the replacement of cylindrical wires by flat strips greatly simplifies the mesh structure for numerical integration.

For the multi-strip—plate system, the collection efficiency as a function of Deutsch number De_{av} and Peclet number Pe_{av}, based on the space-averaged field at the collector E_{av}, agrees closely with the efficiency of a plate—plate collector when E_{av} is identified with the uniform field of the plate—plate collector.

The distributions of particle concentrations are, however, significantly different in the two cases. In particular, results for the strip—plate system show that there are relatively localized regions around each strip electrode (where the repulsive field is very strong) that have very low particle concentrations.

It should be noted that the present study treats the case of the collection of pre-charged particles in a turbulent flow in which both the mean velocity $\bar{u}$ and the turbulent diffusivity D are assumed uniform. However, the formalism presented here is also applicable to the case of specified spatial distributions of $\bar{u}$ and D when sufficient information on these quantities is available from a knowledge of the fluid mechanics.

For the more practical case of a single stage precipitator the situation is further complicated by several factors. First the electric field must be found as a solution of Poisson's equation coupled to the ion flow equation; second, the charging of particles by exposure to the ions must be allowed for; third, the perturbation of the fluid flow by the corona wind may be significant as discussed in [7,8]. Finally, to allow for the effect of corona suppression for high loadings of fine particles, the particle space charge must be included in Poisson's equation, which then becomes coupled to the solution of the convective-diffusion equation.

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