Onset of Flow Reversal and Penetration Length of Natural Convective Flow Between Isothermal Vertical Walls

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Nomenclature

\[ b = \text{channel half-spacing} \]
\[ H = \text{channel depth} \]

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\[ L = \text{channel height} \]
\[ L_p = \text{penetration length of flow reversal} \]
\[ \text{Nu}_b = \text{average Nusselt number based on } b \]
\[ \text{Pr} = \text{Prandtl number} \]
\[ R = \text{radius of extended upstream region from the channel entrance} \]
\[ \text{Ra}_b = \text{Rayleigh number} = \text{Gr}_b \text{Pr} \]
\[ \text{Ra}^* = \text{modified Rayleigh number} = \text{Ra}_b(b/L) \]
\[ x, y = \text{coordinate system for physical geometry} \]

Introduction

Thermally driven flows in a vertical heated channel form a basic structure for various heat transfer devices. Typical applications include electronic packaging systems and heat exchangers. Heat transfer characteristics of the flow to and from the channel walls have been studied extensively, with many studies of channel flows available. The onset and penetration lengths of the flow reversals, however, have received only limited attention. The present study of flow reversals occurring in vertical isothermal channels uses a convenient numerical technique with experimental substantiation by means of smoke visualization.

Since the pioneering work of convective heat transfer phenomena in heated vertical channels conducted by Elenbaas (1942), there have been a number of relevant investigations using various analytical, numerical, and experimental techniques. Eckert et al. (1990) presented an extensive review of publications dealing with natural convection in vertical channel flows. Many different aspects of the problem have been identified and investigated, including local and average correlations of heat transfer coefficients, isotherms, and streamlines for narrow or wide channel flows, symmetrically or asymmetrically heated channels, vertical and inclined arrangements and, more recently, parallel and converging channels (Kihm et al., 1993). These aspects have been studied for forced, mixed, and natural convection problems. There are more studies for forced or mixed convection primarily because of their relative simplicity arising from the parabolic nature of the problems. Natural convection problems, however, are generally more cumbersome because of the nontrivial contribution of transverse and axial diffusion effects, i.e., the elliptic nature of the problem.

Sparrow et al. (1984) investigated the flow reversal by visualizing water flows (Pr = 5.0) in a vertical channel heated on one side. They showed that a single dimensionless group, \( \text{Ra}_b(2b/L) \), where \( 2b \) represents the channel spacing and \( L \) is the channel height, correlated their Nusselt number results well. Chang and Lin (1990) presented a numerical simulation of the transient process of flow reversal in a vertical channel with one side heated. They observed that the velocity and temperature wakes above the heated plate were oscillatory when \( \text{Ra}_b \) was larger than 10³. Naylor et al. (1991) introduced Jeffrey-Hamel flow for the far-field inlet boundary conditions in their numerical study of flow between isothermal walls. Their elliptic solution identified a flow separation near the channel inlet where the Rayleigh number exceeded a critical value, which resulted in a minimal local Nusselt number. As the study focused on the channel inlet region, the flow reversal occurring near the channel exit was not discussed.

The present paper identifies the occurrence of the onset and penetration lengths of the flow reversal in natural convection flow through vertical isothermal channel walls in terms of the modified Rayleigh number \( \text{Ra}^* = \text{Ra}_b(b/L) \) and the channel aspect ratio L/b (Fig. 1).

Analytical and Numerical Study

The standard elliptic forms of the steady two-dimensional mass, momentum, and energy equations were solved using a finite difference method with a body or boundary-fitted coordinate transformation (BFCT). A comprehensive description of
general BFCT technique is given by Thompson et al. (1974) and a
detailed description of the scheme applied to the present problem is
presented by Kim (1993). Laminar flow was assumed with a Boussinesq
approximation. The thermal boundary conditions are isothermal at the
vertical walls, adiabatic at the ceiling of the extended semicircle, and zero
gradient at the extended inlet and channel exit. The boundary conditions
imposed for the velocity field are no-slip on all solid surfaces, including
the ceiling, with Neumann conditions at the inlet and exit.

The half-domain of symmetry consisted of a total of 4961 graded grids
with a 3 percent average expansion or contraction grading. The radius of
the extended region, \( R \), was equal to ten times the channel half-spacing,
\( b \), as preliminary calculations with a setting \( R = 10 b \) converged
within 2 percent of those with a larger radius of \( R = 20 b \). The
BFCT technique successfully transformed a unique configuration
consisting of the channel and the semicircular extended upstream inlet
into a rectangular domain on which the numerical analysis was carried
out. The dimensionless stream function, vorticity, and temperature were
integrated using iterations by successive overrelaxation (SOR) with
convergence of 10\(^{-6}\) or less until the mass continuity was satisfied.

The onset of flow reversal was the value of \( \text{Ra}^* \) that first resulted in a
negative streamline detached from the centerline. Pocketlike streamlines
with negative constants represented the formation of a recirculating flow.
The recirculating flow penetrates into the channel along the centerline or
to zero streamline, splits into two separate flows at the stagnation point and
then each flow merges into the upcoming flow along the channel wall.
The maximum penetration length is the distance from the channel exit to
the stagnation point on the centerline.

Calculations were made for air, \( P_r = 0.7 \), a modified Rayleigh number
range of 0.05 \( < \text{Ra}^* < 2000 \), and channel aspect ratios of \( \lambda/\bar{b} = 2, 5, 8, 10, \) and 24. The calculated heat transfer coefficients showed excellent
agreement with previous data (Kim, 1993). The average Nusselt number,
\( N_u \), correlated well with the modified Rayleigh number \( \text{Ra}^* = \text{Ra}_{\lambda} (b/\bar{b}) \).

**Flow Visualization Study**

An experimental test facility for the flow visualization of recirculating
flow behavior in isothermal vertical channels was developed. Each of the
vertical isothermal walls was made of 0.826-cm-thick aluminum plate
with a vertical length, \( \lambda \), of 12.7 cm and a depth, \( \bar{b} \), of 20.32 cm. Each
plate had a heater

![Fig. 1 The computational domain for convective air flow in vertical isothermal channel configuration](image)

**Results and Discussion**

Figures 2(a, b, c) show the numerically calculated streamlines (left side) and isotherms (right side) for \( \text{Ra}^* = 1, 100, \) and 500 for the
case of \( \lambda/\bar{b} = \bar{b} \), respectively. The inset photographs were
taken from the smoke visualization experiment of the flow stream at
\( \text{Ra}^* = 130.2 \) and 501.8. The extended inlet regions are shown only up to
twice the channel half-spacing. The dimensionless isotherms have a value
of one at the isothermal wall and zero at free stream and infinity. The
dimensionless stream line takes a value of zero at the centerline. For the
case of \( \text{Ra}^* = 1 \) (Fig. 2a), the almost uniform intervals between streamlines show that the flow is fully developed. The weak air flow
induced by the low thermal driving force allows the isotherms to extend
well into the upstream entrance region due to the elliptic nature of the
relatively strong conduction effect. As a result, the incoming air is
preheated and the temperature at the channel inlet can be as much as 30
percent higher than the ambient or far-field inlet temperature. Thus, a
parabolic simplification assuming uniform inlet temperature would cause
a significant error for such low Rayleigh number cases.

The flow field at \( \text{Ra}^* = 100 \) (Fig. 2b) deviated from the previously fully
developed case and the depleted streamlines near the channel center show
separate boundary layer development.
The more concentrated streamlines appearing near the wall suggest increased buoyancy-driven air flow. Also, this increased convection reduces the penetration of the conduction effect at the inlet as seen in the isotherm distributions.

Figure 2 (c) shows the streamlines and isotherms when \( \text{Ra}^* \) is increased to 500, which exceeds the critical Rayleigh number (\( \text{Ra}^* = 146 \) as numerically determined). The congested streamlines near the wall indicate that most induced air flow concentrates in the narrower boundary layer next to the wall. The pocketlike streamlines appear with negative constants demonstrating the formation of recirculating flow. Vena-contract-like streamlines at the entrance appear to reduce the effective opening. This narrower effective opening reduces the incoming air, whereas the increased thermal driving force requires more air flow. When the acceleration of the air flow near the channel wall exceeds a critical point, the incoming air flow through the inlet is insufficient and additional air drawn in through the central portion of the channel exit results in flow reversal. The

![Image](image_url)

**Fig. 2** Calculated flow streamlines and isotherms of convective air (Pr = 0.7) for \( L/b = 8 \): (a) \( \text{Ra}^* = 1.0 \); (b) \( \text{Re}^* = 100 \); and (c) \( \text{Re}^* = 500 \). The photographs indicate smoke stream visualization at \( \text{Ra}^* = 130.2 \) and \( 501.8 \), respectively.

The isotherms show that the temperature in the recirculation region is significantly lower than the air temperature next to the wall at the same \( y \) position. This indicates that the convection heat transfer to the recirculating air from the wall is limited and the overall heat transfer is not significantly altered by the presence of the flow reversal. The fact that the conduction preheating of the incoming air, or the elliptic nature of the problem, has noticeably been reduced with the occurrence of flow reversal, implies that the channel flow problem can be approximated with a parabolic simplification when \( \text{Ra}^* \) exceeds the critical value for the flow reversal.

Figure 3 shows that the calculated onset Rayleigh numbers that initiate the flow reversals are 120, 140, 146, 180, and 450, respectively, for \( L/b = 24, 10, 8, 5, \) and 2. The numerically and experimentally observed penetration lengths of the channel flows are also provided in Fig. 3. Although the present experimental configuration allowed only two channel aspect ratios, 8 and 10, the data favorably supported the numerical results. \( \text{Ra}_b \) is the flow parameter that determines the level of natural convection strength and \( L/b \) is the geometric parameter. It appears that the penetration length increases for increasing channel aspect ratio, \( L/b \), for the same \( \text{Ra}^* = \text{Ra}_b (b/L) \). This is primarily because of higher flow parameter \( \text{Ra}_b \) required to keep \( \text{Ra} \) constant as the channel is longer with smaller \( b/L \). This higher \( \text{Ra}_b \) also explains the fact that the onset of the flow reversal for longer channel occurs earlier at relatively smaller \( \text{Ra}^* \). Another thing to note here is that the deviation observed between the data and predictions particularly in lower \( \text{Ra}^* \) range is probably attributed to the simplified free boundary condition imposed at the channel exit. For further investigation of the problem, a downstream extended region with a far-field free boundary condition needs to be considered to incorporate the possible elliptic behavior at the channel exit.

**References**


