Optical tomography using a genetic algorithm

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A new tomographic image reconstruction method is proposed that uses a genetic algorithm (GA), a robust optimization algorithm based on the genetic principle of natural selection. For the purpose of description, a simple axisymmetric reference density field is reconstructed from its interferometric projection by the developed GA-based tomography. This preliminary investigation shows a promising potential of the GAbased tomography to overcome the problems associated with other existing tomographic methods, particularly for limited projections. © 1996 Optical Society of America

One crucial issue in tomographically reconstructing an actual field from a set of optically projected images is how to maximize the reconstruction accuracy given a minimal number of projections. Conventional tomography schemes belong to either of two major categories: a direct mathematical inversion calculation of the cross-sectional field from projected images or a successive iteration of the field by means of regression optimization. The former category, typically called a Fourier convolution, is based on the Fourier slice theorem[1] and usually requires many equally angled projections to ensure its mathematical stability and acceptable accuracy. Spatial and other constraints, however, often limit the number and angles of projections, and thus the reconstruction accuracy can be degraded, particularly for the case of limited projections.[2] The latter category, represented by the algebraic reconstruction technique,[3] usually needs fewer projections for a stable reconstruction than do the Fourier convolution techniques. However, the algebraic reconstruction technique is a monotonic optimization, and once the iteration converges to a local peak the solution tends to lock without further searching for higher peaks. Furthermore, when the number of iterations exceeds a certain optimum, the ill-posedness of tomographic reconstruction using the algebraic reconstruction technique can increase artifacts and destroy the reconstruction.[4]

We discuss an innovative tomographic reconstruction scheme using a genetic algorithm[5] (GA), a combinatorial and robust function optimization based on the mechanics of genetic principles. The combinatorial feature of the GA is well represented by a population of solution candidates, instead of one evolving toward a solution. The mutating feature of the GA permits the unique robustness that prevents the algorithm from being entrapped by a local peak. The key idea is that these features of the GA can be used to establish a new tomographic method that can alleviate those problems associated with the existing techniques. Although the idea is applicable to any optical projection method, an interferometric projection of an axisymmetric density field has been selected solely for demonstrative purposes herein.

We can see the mathematical features of the GA by looking at an example of an axisymmetric density field with 10 discrete nodal points (Fig. 1). The axisymmetric density field is to be reconstructed from a single projected interferometric fringe image at any projection angle. Although the goal is to find one optimum set of (ρ_1, ρ_2, ρ_3, ..., ρ_{10}), the GA initiates and proceeds with a population of N individuals in which each individual carries randomly assigned initial guesses for the field: (ρ_1^i, ρ_2^i, ρ_3^i, ..., ρ_{10}^i) for $i = 1, 2, \ldots, N$. Currently there are no accepted rules of thumb for choosing the number of individuals in the population. However, the nature of regression anticipates that the presence of more individuals in the population requires fewer evolving generations for similar convergence. The density values at the discrete nodal points, ρ_1^i, ρ_2^i, ρ_3^i, ..., ρ_{10}^i, are treated as genes of the *i*th individual. Successive generations evolve following the genetic principles of natural selection and survival of the fittest that constitute the basis of a scaled random principle.

The rms discrepancy of each individual's projection from the measured projection of the actual field is called the fitness value of the individual. Thus a smaller fitness value is given to an individual with smaller deviation. A pair of individuals is then selected based on fitness rankings in which an individual's chance of being selected is proportional to the reciprocal of its fitness value relative to those of the other individuals in the population. This selected pair of individuals has a specified probability

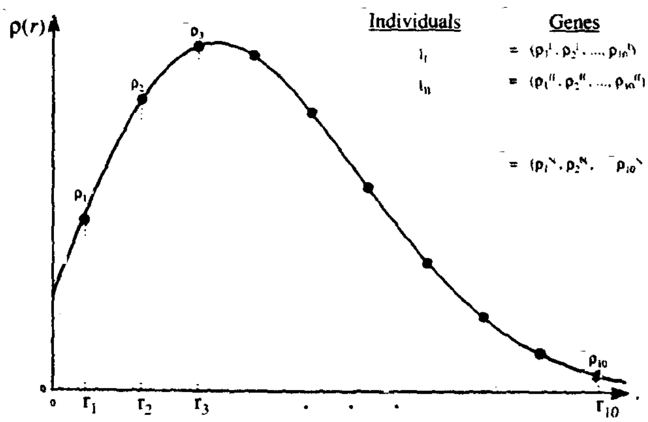

Fig. 1. Axisymmetric density field to be reconstructed with 10 nodal points.

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(50% in the present study) of creating two children through swapping or crossover of some of their genes.

The children will replace their parents, and all their genes have a specified probability (currently 1%) of a mutation. Mutation is achieved when the value of a gene is changed a randomly specified small amount from the current value. The primary role of gene mutation is to prevent the population from settling at a locally converging peak and to provide a robust optimization in searching for the ultimate peak. Any twin or triplet individuals will count as only one to avoid the potential of dominance by common. A new generation evolves from the set of procedures of selection, crossover, mutation, and fitness evaluation until the projection of the first-ranked individual with the highest fitness (smallest fitness value) falls within a specified convergence from the measured projection of the actual field or, alternatively, until a specified number of generations is reached.

An elementary distribution function $f(r-r_1, A_i, t_i)$ replaces the discrete density value at each nodal point to enhance the spatial resolution of the reconstructed field without significantly increasing the computational expense (Fig. 2). A_i and t_i represent the magnification and shape factors, respectively. The density field is assumed to conform to the summation of all the elementary distribution functions, i.e., $\rho(r)=\sum^{10}_{i=1} f(r-r_i;A_i,t_i)$. Now the reconstruction problem reduces to finding an individual carrying optimized genes of $(A_1, A_2, ..., A_{10}, t_1, t_2, ..., t_{10})$. We search for this solution by using the previously discussed genetic algorithm procedures of selection, crossover, mutation, and fitness evaluation.

Although the number of unknowns has increased by a factor of 2 in the example (20 from 10), when elementary distribution functions (EDF's) replace the discrete density values the spatial resolution of the desired density field improves to that of a continuous function. Another thing to note is that, as each EDF extends over the entire field, the necessary number of EDF's to describe the field is expected to be much fewer than the number of nodal points required for discrete unknowns. In principle, the EDF's can be any type of continuous function, such as a Gaussian function, a Rayleigh function, a sine or a cosine function, or a mass diffusion solution. Proper care in selecting a function type for EDF to represent best the physical conditions of the tested field may expedite the reconstruction optimization.

Two independent parameters, the magnification and the shape factors, determine the EDF considered in the present study, whereas a determination of a single magnification factor for the conventional basis function completed the solution search in most other regression methods.[6] Existing regression methods are not able to optimize two independent parameters because these methods are somewhat monotonic and lack multidimensional features. On the other hand, the number of parameter sets to be optimized does not limit the scope of the GA-based algorithm.

To demonstrate the utility of the GA approach to tomography, a preliminary calculation was carried out for a mathematically specified axisymmetric reference density field shown as the bold curve in Fig.3:

$$\rho(r) = K_a \exp(-K_b r^2) + K_c r \exp(-K_b r^2) \qquad (1)$$

where K_a, K_b, and K_c, are scaling constants for normalization of the density field. An interferometric projection function' has been selected for the current example, and a total of 10 Gaussian EDF's, A $\exp[-(r - r_i)^2/t_i]$, have been defined at equal intervals along the radius. The best individual from the (randomly assigned) initial generation of 100 individuals shows large deviations from the reference field. However, the discrepancies have quickly decreased as the generations have evolved, and the reconstructed density field from the 100th generation has already approached the reference field in close proximity.

Figure 4 shows the history of convergence along the evolving generation. The convergence of the average and the best individuals has reached the asymptotic level near the 500th generation. Although the average fitness fluctuates temporarily, the fitness of the best individual at least maintains its current level or improves with generations. This is due to the elitism imposed in the GA in that the current best maintains

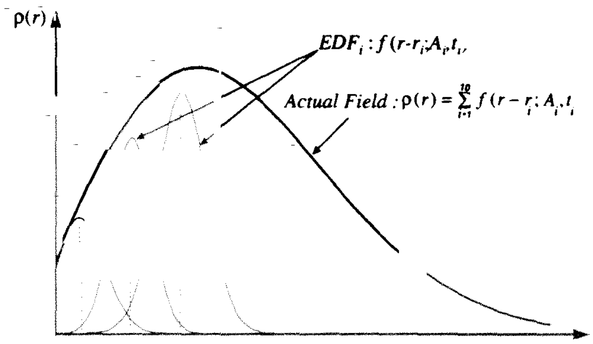

Fig. 2. Axisymmetric density field to be reconstructed with 10 EDF's.

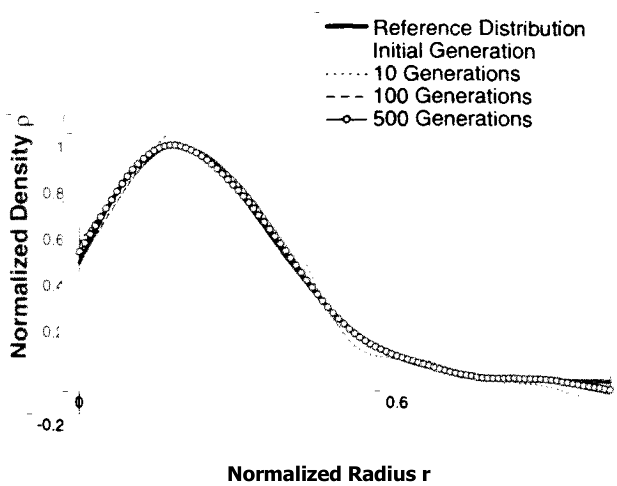

Fig. 3. Evolution of the reconstructed density field with evolving generations.

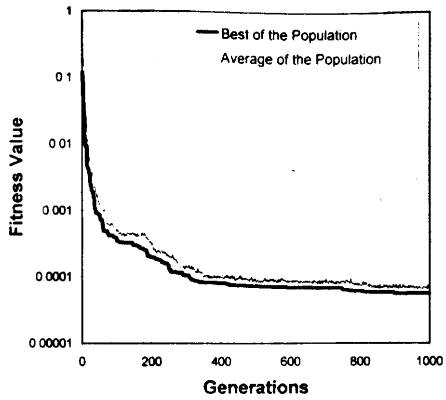

Fig. 4. History of fitness value with evoling generations.

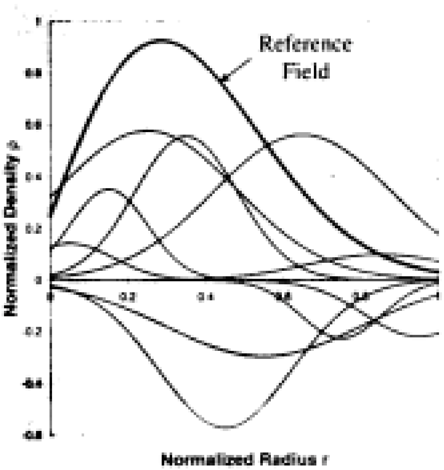

Fig. 5. Optimized EDF's conforming to the reconstructed density field at the 1000th generation.

the title until a better individual with a smaller fitness value in a future generation captures it.

Figure 5 shows the individual EDF's that constitute the density field at the 1000th generation. Whereas it might be expected that the individual EDF's would have particular characteristics in certain areas of the density field, i.e., relatively narrow and tall near the left end and short and broad toward the right end, it must be remembered that it is the sum of the EDF's that determines the overall density field. Also, the negative EDF's compensate for the excessive contributions from the positive EDF's, which facilitates a more effective optimization process. Focusing solely on the fitness of individual solution candidates, the ability of the GA to proceed toward the optimum solution without any preconceived ideas of what a good solution should be like gives the genetic algorithm superiority over conventional optimization techniques, especially for conceptually difficult problems or nonalgebraic imaging such as laser speckle photography.[8]

Although this Letter has focused on the GA applied to an example reconstruction of an axisymmetric field, studies in progress for the reconstruction of an asymmetric density field show equal success. The GA-based tomography shows promise, particularly for limited projections. A study is currently being conducted to examine systematically the effects of the GA parameters, such as the number and types of EDF's and the probability of crossover and mutation, on the reconstruction accuracy. The authors plan a rigorous merit comparison of the GA tomography with upto-date existing techniques such as the multiplicative algebraic reconstruction technique, singular value decomposition, and various other transform algorithms.[9]

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