Laser Speckle Photography Technique
Applied for Heat and Mass Transfer Problems

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I. Introduction

Speckle photography technique [1] is an optical diagnostic method to measure nonintrusively the refractive index gradients of inhomogeneous medium, also called a phase object, based on light ray refraction and deflection. The measured refractive index gradients are converted into the medium temperature and density information using pertinent physical equations. The technique provides highly accurate and quantitative information on the temperature and density fields, with an excellent spatial resolution.

The encyclopedic definition of optical speckle is “a phenomenon in which the scattering of light from a highly coherent source, such as a laser, by a rough surface or inhomogeneous (refractive index) medium generates a random-intensity distribution of light that gives the surface or medium a granular appearance [2].” Here, the random-intensity distribution results from the randomly constructive or destructive interference of the scattered light rays.

Speckle photography is frequently confused with particle image Velocimetry (PIV) under dense seeding particle concentration [3]. However, specklelike images of PIV are the result of overlapping of Mie scattering images from the physically seeded particles and not from optically created speckles. The dislocations of speckle images result from the flow-particle displacement in PIV. In this chapter, speckle photography refers to an optical method that measures the refractive index gradients of a phase object based on the optically created and refractively dislocated speckle patterns.
Part II introduces the operating principle of the speckle photography technique and discusses its unique features compared with other competing optical methods.

In Part III, several different natural heat convection problems are presented for which speckle photography successfully measures the heat transfer coefficients without the need of corrections for the conduction and radiation losses. Speckle results provide data with high spatial resolution and make the experimental processes more reliable.

Part IV shows that speckle photography measures the statistical properties for turbulent flows with density and temperature fluctuations. Also discussed is tomographic reconstruction of a density field from multiple specklegrams.

The potential of speckle photography for high-temperature applications such as combusting flames is discussed in Part V. The viability of the technique emerges from its applications for premixed Bunsen flames of axisymmetric and laminar configuration.

Finally, Part VI brings an example application of speckle photography for a liquid flow with density variation where the refractive index behaves quite differently from that of air or other gases. Part VII closes with concluding remarks.

II. Speckle Photography Technique

Section A discusses the basic principle of laser speckle photography and its advantages over interferometry. In Sections B and C, two alternative data processing methods for specklegrams are presented. Section D discusses speckle interferometry as a variation of speckle photography. A brief comparison of speckle photography is given and speckle interferometry is discussed in Section E.

A. OPERATING PRINCIPLE OF SPECKLE PHOTOGRAPHY

The early stage of speckle photography attempted to use incoherent white light that allowed only a qualitative visualization of the medium index field [4]. The use of a laser for the system made speckle photography a truly quantitative and rigorous measurement tool [5, 6]. This section summarizes the basic principles of a modern laser speckle-photography system [7].

The randomly deflected and interfered light rays from spatially distributed optical speckles beyond the ground glass (optical deflector), as illustrated in Fig. 1(a). A bright optical speckle forms for coherent or constructive ray interference and a dark speckle results from incoherent or destructive interference. Diffraction laws [1, 8] give the size of an individual speckle as \( d = 2.44 \lambda \cdot H/D \), where \( \lambda \) is the wavelength of the laser radiation; \( H \) is the distance of the observation point from the deflector, that is, the ground glass surface; and \( D \) is the dimension on the grain size of the ground glass plate. Li and Chiang [9] presented a detailed analysis for calculating the fully three-dimensional (3D) dimension of laser speckle formation.

When a test section is presented, the light rays refract at a small angle because of the variable density or refractive index field, and the corresponding speckles form at different locations (Fig. 1(b)). A specklegram is taken by photographically overlapping the original speckles without a test section, and the dislocated speckles with a test section. The resulting speckle photograph, or specklegram, carries the information on both sizes of the speckle images and the amount of the speckle dislocations.

The optical layout of the speckle photography system closely resembles that of a Schlieren system [10]. Figure 2 shows the setup of the speckle photography system configured by Kastell et al. [11]. A helium-neon laser beam of \( \lambda = 632.8 \text{ nm} \) expands to 120 mm in diameter and collimates via the first parabolic mirror to form parallel light rays throughout the test section. After passing through the test section, the light rays reflect from the second parabolic mirror...
and collimate its focal point. After the focal point, the light rays illuminate on the optical-grade ground glass plate (Fig. 3(a)) whose average grain size is on the order of one micron [12]. Commercial ground glass with coarser grains (Fig. 3(b)) does not generate speckles with acceptable resolution for optical measurement.

![Diagram](image1.png)

![Diagram](image2.png)

Laser speckles also can be generated by using a diffuser plate sprayed with an acrylic paint sealant [13]. However, the random phase of the diffuser plate is usually coarser than the sandblasted ground glass plate. Since the speckle resolution depends on the scale of the surface grains, the ground glass plate is preferred for its finer grain size.
Fig. 3. Various diffracting surfaces under × 200 magnification. (a) Experimental grade ground glass; (b) commercial grade ground glass; (c) 500 × 50 copper wire mesh [12].
The speckle dislocation due to the presence of an inhomogeneous test field is schematically illustrated in Fig. 4. Without going through a test field, the light forms a speckle at the location I on the image plane. When the test field is present, the light ray refracts at a small angle $\alpha$ because of the density variation, and forms another speckle at $I'$ on the specklegram. From the geometrical similarity, $\varepsilon = \alpha/m$ where the magnification of the second parabolic mirror $m$ is equivalent to the ratio of the image distance to the object distance, that is, $m = b/a$. The speckle dislocation $\Delta$ is then expressed as

$$\Delta = m' \times \delta = m' \times c \times \varepsilon = \frac{m'}{m} \times c \times \alpha \quad (1)$$

where $m'$ is the magnification of the camera lens, and $\delta$ the amount of speckle dislocation on the focusing plane for speckle imaging. The distance between the ground glass and the focal plane of the camera lens is denoted by $c$. This distance is called the defocusing distance.

The test section image is focused onto the ground glass. However, the camera focusing plane for the speckle image recording must be slightly away from the ground glass, because the speckle dislocations are not detectable on the ground glass itself (Fig. 1). The defocusing thus is inevitable to record a detectable dislocation of speckle images. Nevertheless, the defocusing distance must be minimized to reduce image blur. In estimating the optical defocusing distance, we must consider both optical parameters and test conditions. As a rule of thumb, however, about 2 cm of defocusing is recommended for the initial attempt at recording specklegrams [14]. The analysis for calculating the image blur in specklegram recording has been developed based on geometrical ray optics [15]. The analysis suggests that the test object be placed near the optical axis to reduce the blur and that mirrors of larger diameters with longer focal lengths enhance the measurement sensitivity of the speckle photography system.

The main advantages of speckle photography over conventional optical methods relying on ray refraction, such as wave front interferometry, the Schlieren method, and holography, can be summarized as follows [16]:

1. The optical setup is simple because of the single beam path that needs no reference beam
2. Requirements concerning mechanical stability against the external disturbance are less stringent
3. Specklegrams have higher spatial resolution of information density than do interferograms
4. Speckle photography using a short pulsed laser can effectively record turbulence flow properties, whereas interferometry fails to construct any stable fringes for turbulent flows.

The most striking advantage of speckle photography over conventional methods is that quantitative information on the refractive index (or its gradient) is obtained along each beam trajectory, independent of one another, without the need to
know the values of adjacent beams. This important feature of the speckle method makes it suitable for turbulent flow studies, whereas conventional methods fail because they must be interpreted simultaneously over the entire field.

B. POINT-BY-POINT INTERROGATION OF A SPECKLEGRAM

The point-by-point interrogation [17] of a specklegram determines the distance of the speckle dislocation \( \Delta \) from the spacing of Young's fringes that are generated by illuminating the specklegram with a low-power laser (Fig. 5). The principle of Young's fringes [18] states that the fringe spacing \( s \) decreases with an increase of the speckle dislocation \( \Delta \); that is,

\[
    s = (s_y, s_z) = \frac{\lambda \times d}{\Delta}
\]

where \( s_y = s \cos \theta \) and \( s_z = s \sin \theta \), with \( \theta \) being the tilt angle of the fringe orientation on the \( y-z \) recording plane; \( \lambda \) is the wavelength of the interrogating laser beam (\( \lambda = 632.8 \) nm for a helium-neon laser); and \( d \) is the distance between the interrogating specklegram and the imaging screen.

A charge-coupled device (CCD) camera digitizes the fringe images for a computerized evaluation of the fringe characteristics [19]. The uncertainties associated with manually reading fringe spacings have been reported to be as high as \( \pm 15\% \) in some cases [20]. Erbeck [21] has developed a more sophisticated specklegram processing system that uses a rotating ground glass screen to enhance the signal-to-noise ratio (SNR) in reading fringe spacings (Fig. 6). The rotating ground glass screen reduces the fluctuating small-scale speckle noise. The computerized analysis, using an advanced fringe reading software, is able to reduce the uncertainties below \( \pm 3\% \). Less popular alternative techniques for interrogating specklegrams are presented elsewhere [22-24].

![Fringe Image Plane](image)

**Fig. 5.** Point-by-point data analysis by measuring Young's fringes generated from specklegram.

Within the context of geometrical optics, Fermat’s principle describes the propagation of a single light ray through a medium and states that the light wave takes a path that involves the least travel time. The use of a paraxial approximation, that is, the amount of ray diffraction in the medium being very small and the light ray remaining nearly straight (Fig. 7), the diffraction angle components \( \varepsilon_y \) and \( \varepsilon_z \) are expressed as the path integral along the optical axis (25):

\[
    \tan \varepsilon_y \equiv \varepsilon_y(y, z = z_k) = \int_{x_1}^{x_2} \frac{1}{n(x, y)} \frac{\partial n(x, y)}{\partial y} dx, \quad (3a)
\]

\[
    \tan \varepsilon_z \equiv \varepsilon_z(y, z = z_k) = \int_{x_1}^{x_2} \frac{1}{n(x, y)} \frac{\partial n(x, y)}{\partial z} dx. \quad (3b)
\]
Equations (3a) and (3b) describe the $x$-$y$ cross section at $z = z_k$. Repeated slicings at different $z$ coordinates describe the whole 3D field. The $y$-$z$ recording plane is perpendicular to the optical axis $x$, and the light ray occupies the test field between $\xi_1$ and $\xi_2$ along the optical axis.

Substituting Eqs. (1) and (2) into Eqs. (3a) and (3b) constitutes the basic relations between the measured fringe spacing and the medium refractive index field:

$$ s_y(y, z_k) = \frac{m \lambda d}{m^2 c} \left[ \int_{\xi_1}^{\xi_2} \frac{1}{n} \frac{\partial n}{\partial y} \, dx \right]^{-1}, $$

$$ s_z(y, z_k) = \frac{m \lambda d}{m^2 c} \left[ \int_{\xi_1}^{\xi_2} \frac{1}{n} \frac{\partial n}{\partial z} \, dx \right]^{-1}. $$

which depict that the fringe spacing reflects the line-of-sight effect from an integrated contribution of the refractive index field along the occupied laser ray. The integral expressions of Eqs. (4a) and (4b) must be inverted to determine the refractive index field distribution on the tomographic interrogating plane.
C. FULL-VIEW INTERROGATION OF A SPECKLEGRAM

An alternative reconstruction method is a full-view semiquantitative decoding by spatial filtering of a specklegram [26]. Figure 8 shows an optical arrangement for spatial filtering where the specklegram is illuminated with an expanded collimated beam. A Fourier transforming lens (the second lens) produces the spatial Fourier transform of the speckle patterns in the back focal plane of the lens. A small and eccentric filtering aperture placed at the focal plane selects the diffraction order and direction of speckle displacement. A fringelike pattern can be reconstructed on the imaging screen, as shown for the isothermal vertical wall in Fig. 9 [27]. The visible patterns indicate constant density or temperature gradient contours, and these patterns are analogous to the conventional interferometric fringes in the infinite fringe width mode [28]. Although the resolution and accuracy are low for accurate quantitative information, the spatial filtering method is useful for a quick qualitative check of the recorded flow situation prior to the more time-consuming point-by-point evaluation of Young’s fringes.

![Fig. 8. Optical spatial filtering of a specklegram: The range of visible spatial frequencies determined by the position of the hole [26].](image)

![Fig. 9. Whole-field reconstruction of a laminar, natural convection flow along an isothermal vertical wall [27].](image)

D. SPECKLE INTERFEROMETRY

Speckle interferometry [29], an alternative visualization method using optical speckles and their dislocations, is a hybrid system of speckle photography and interferometry, as schematically shown in Fig. 10. Optical speckles are essentially generated by passing the object beam through a ground glass, as in speckle photography. However, speckle
interferometry devises an additional reference beam that eventually merges with the object beam via the half-reflecting mirror. A slight variation of speckle interferometry is a double-hole aperture that can substitute for the reference beam (Fig. 11). This optical arrangement is called speckle shearing interferometry, where the speckle images constructed by the two holes are separated, sheared, and eventually imaged together on the recording plane [30]. This is analogous to the beam separation and merging of interferometry. Speckle shearing interferometry generates interferograms similar to those generated by speckle interferometry. Therefore, these fringes can be read in the same way as for interferometry, with alternate dark fringes indicating a 2-π phase shift equivalent to a single wavelength differential in the optical path.

Both speckle interferometry and speckle shearing interferometry are double-exposure techniques, one with and one without the presence of a test field. The local brightness of speckles depends on whether the combination of the two speckles are coherent or incoherent. The combination of the two speckle patterns results in a fringe pattern on the photographic negative. Thus, by evaluating the fringe pattern, information regarding the changes in the refractive index between the two exposures is obtained.

Viewing these fringes with a higher SNR can be achieved by the whole-field filtering of the speckle interferogram, which is similar to the whole-field filtering of speckle photography (Fig. 8). Figure 12 shows a typical shearing interferogram of gaseous Bunsen flame obtained after the whole-field filtering [31].
E. COMPARISON OF SPECKLE PHOTOGRAPHY VERSUS SPECKLE INTERFEROMETRY

Speckle interferometry and speckle shearing interferometry provide information regarding the refractive index itself, while speckle photography provides the gradient of refractive index. Although speckle (or speckle shearing) interferometry offers the convenience of providing a full-field fringe pattern that allows both a qualitative inspection and semiquantitative results, the technique has all the disadvantages of interferometry (Section A). Speckle interferometry was found to be easier than speckle photography in some applications for direct temperature measurements [32]. Nevertheless, speckle interferometry needs a priori information on the direction of temperature gradient to complete the quantitative data analysis. It also requires a photographic film with higher resolution compared with speckle photography. For these reasons, speckle photography is more popular for thermal and fluid applications. The subsequent chapters will primarily discuss the applications of speckle photography.

III. Applications to Natural Convection Problems

Laser speckle photography has been applied for natural convection problems where the refractive index gradient is strong only in a single direction, for example, in the y-direction in Fig. 7. The refractive index gradient is zero along the ray parallel to the x-direction. A typical example that satisfies these two conditions is a vertical isothermal wall aligned parallel to the x - z plane.

Section A presents a speckle data analysis for one-dimensional (1D) ray refraction, while Section B discusses the application of speckle photography for a natural convection heat transfer coefficient. Section C shows applications for natural convection flows along an isothermal vertical wall, Section D for thermal boundaries along an isothermal vertical channel, and Section E for a horizontal isothermal plate. Finally, Section F discusses the heat transfer and boundary layers for converging channels with different inclination angles.

A. SIMPLIFIED DATA REDUCTION FOR ONE-DIMENSIONAL REFRACTION

For such a simple geometry allowing 1D refraction, such as a vertical isothermal wall, Eq. (4b) is zero and Eq. (4a) reduces to

\[
\frac{\partial n}{\partial y} = \frac{m}{m'} \cdot \frac{\lambda d}{cL} \cdot \frac{1}{s_y}
\]  

(5)
where \( L \) denotes the dimension of the test field measured along the optical axis. The chain rule of differentiation converts Eq. (5) into an expression for a temperature gradient as

\[
\frac{\partial T}{\partial y} = m \frac{\lambda d}{m' c L} \frac{1}{s_y} \left( \frac{\partial n}{\partial T} \right)^{-1}
\]

where the refractive index change per unit temperature, \( \partial n/\partial T \), for air at the 632.8 nm wavelength of a helium-neon laser, is [28]

\[
\frac{\partial n}{\partial T} = \frac{1.075 \times 10^{-6}}{(1 + 0.00368184 T)^2}
\]

where \( T \) is in degrees Centigrade. The use of the ideal gas equation of state, together with the Gladstone-Dale relationship [33], \( n = 1 + K \rho \), with \( K \) being the Gladstone-Dale constant (for air 0.2257 x 10^{-3} m^3/kg at \( \lambda = 632.8 \) nm), converts Eq. (5) into an expression for the density gradient, that is,

\[
\frac{\partial \rho}{\partial y} = m \frac{\lambda d}{m' c L} \frac{1}{s_y} \frac{1}{K}
\]

A high refractive index gradient increases the speckle dislocations and decreases the fringe spacing \( s_y \) (Eq. (5)). A small fringe spacing reflects a large temperature or density gradients (Eq. (6) or (8)). Another point is that the fringes are perpendicular to the average direction of the speckle dislocations, which is parallel to the primary direction of the refractive index, temperature, and density gradients.

Figure 13 shows the fringe patterns at five locations inside and outside the thermal layer of an isothermal vertical wall [11]. Near the wall where the temperature gradient is very steep, the fringes are very definite and the spacings are narrow (inset photograph (d)). The fringe spacing becomes wider as the edge of the boundary layer is approached (inset photograph (e)). The fringes disappear outside the thermal layer where there is no detectable dislocation of speckles (inset photograph (b)). The orientation of the fringes around the leading edge is altered because the fringes (or the primary direction of speckle dislocation) must appear perpendicular to a local temperature gradient (inset photographs (a) and (c)).

B. DETERMINATION OF THE HEAT TRANSFER COEFFICIENT USING SPECKLEGRAM DATA

The heat transfer to or from a solid surface can simulate many popular engineering problems, for example: cooling in electronic packaging, turbine blade cooling, and a variety of heat exchangers. Assuming a steady problem with uniformly distributed heat sources, such as a plate heater affixed to the solid base shown in Fig. 14, the energy balance yields

\[
\dot{q} = \dot{q}^\text{rad} + \dot{q}^\text{cond} + \dot{q}^\text{conv}
\]

One traditional measurement technique for heat convection involves setting the total heat generation \( \dot{q}^\text{total} \) and then determining the heat radiation \( \dot{q}^\text{rad} \) and the conduction losses \( \dot{q}^\text{cond} \). These last two terms are subtracted from \( \dot{q}^\text{total} \) to determine \( \dot{q}^\text{conv} \) and \( h \); that is,

\[
h = \frac{\left( \dot{q}^\text{conv} = \dot{q}^\text{total} - \dot{q}^\text{rad} - \dot{q}^\text{cond} \right)}{T_v - T_{\infty}}
\]

This method requires that the whole experimental apparatus-other than the test surface-be wrapped with bulky insulation material to reduce the losses to the environment, which are not easy to estimate accurately. The uncertainties in determining the radiation heat losses are equally difficult to quantify. Incorrect evaluation of these heat losses can significantly affect the measurement accuracy and uncertainties, especially for natural convection problems for which the heat loss amounts can compare with the amount of convection heat transfer itself.
Such difficulties are alleviated by directly measuring the heat transfer coefficient \( h \), without involving the total heat generation rate. At the wall, the velocity is zero and the heat transfer into the fluid takes place by radiation and conduction; that is,

\[
\dot{q}_\text{total} = \dot{q}_\text{radiation} + \dot{q}_\text{conduction}
\]  

(11)

The heat transfer into the fluid away from the wall can be expressed as

\[
\dot{q}_\text{total} = \dot{q}_\text{radiation} + \dot{q}_\text{convection}
\]  

(12)

Combining Eqs. (11) and (12) with Newton's law of cooling [34], the convection heat transfer coefficient is given as

\[
h = - \left( \frac{\partial T}{\partial y} \right)_{\text{wall}} \frac{k}{T_w - T_m}
\]  

(13)

where the \( y \)-coordinate is normal to the heater surface. The heat transfer coefficient can be determined once the wall temperature gradient \( (\partial T/\partial y)_{\text{wall}} \) is provided from the measurement.
The laser speckle photography technique makes it possible to find the heat transfer coefficient by evaluating the temperature gradient at the solid surface from the measured fringe spacing data (Eq. (6)). The wall temperature and ambient temperature are measured by a traditional means such as a thermocouple probe. Use of the laser speckle photography technique significantly reduces the experimental uncertainties associated with the heat loss corrections and provides a noticeably improved convenience in performing experiments [35].

C. ISOTHERMAL SINGLE VERTICAL WALL

The vertical heated plate has been the standard for comparing different optical methods, particularly since its well-established analytical and numerical solutions are available for the temperature profile [36-40]. The viability of laser speckle photography has been examined by applying it to the isothermal vertical wall shown in Fig. 13. Using Simpson’s rule [41], the temperature profiles were calculated by numerically integrating the temperature gradient data obtained from the fringe spacing results (Eq. (6)).

Temperature data at $x = 14.2$ mm and $x = 50.0$ mm are presented in Fig. 15 for several different wall temperature conditions. In this presentation, the kinematic viscosity of air was evaluated at the reference temperature $T_r = T_w - 0.38(T_w - T_e)$, as suggested by Sparrow and Gregg [42]. The similarity solution by Ostrach [43] assumed an ideal thermal boundary layer starting exactly at the leading edge, while the practical boundary layer starts before the leading edge and grows thicker than the ideal one [44]. This thicker boundary layer makes the measured temperature data higher than the theoretical predictions, particularly in the region near the leading edge at $x = 14.2$ mm.
The solid symbols in Fig. 15 represent developed temperature profiles away from the leading edge at \( x = 50.0 \) mm, where the leading edge effect has been diminished. Also presented is a set of data obtained at the same \( x \) location by Schmidt and Beckmann [45], who used a miniature thermocouple sensor probe to measure temperature. The speckle data provide higher measurement resolution than does the thermocouple probe and show good agreement with the theory.

Shu and Li [27] used an optical system to record a specklegram and a shearing interferogram (Fig. 16) simultaneously to visualize an isothermal vertical wall. This combined system used a pulsed ruby laser as the light source for high-speed data recording. The interferogram gives a qualitative examination of the field, and the fringe analysis [28] of the interferogram provides temperature data. The two measurements showed close agreement in the temperature profile for the region away from the leading edge.

Wernekink and Merzkirch [46] conducted a more rigorous comparison. Figure 17 shows an excellent comparison of data obtained by the speckle photography technique, a shearing interferometer using a Wollaston prism, a Mach-Zehnder interferometer, and Schlichting’s similarity results [47]. The results are plotted as dimensionless temperature \( \theta = (T - T_\infty)/(T_w - T_\infty) \) versus normalized \( y \)-distance from the wall, \( \eta = y/\delta \), with \( \delta \) being the boundary layer thickness. The data for the three methods were taken at the region away from the leading edge, where the flow similarity was satisfied. The data showed good agreement with the similarity solutions.

![Fig. 16. Optical system for simultaneous recording of a specklegram and a shearing interferogram of a phase object [27]. He-Ne, helium-neon.](image)

![Fig. 17. Nondimensional temperature \( \theta \) in the laminar, free-convective flow along a vertical, heated plate, plotted against the nondimensional distance from the wall \( \eta \) [46].](image)
D. ISOTHERMAL VERTICAL CHANNEL FLOWS

Speckle photography can visually determine the length of the developing region of the thermal boundary layers growing along the isothermal vertical channel walls [48]. Figure 18 shows the Young's fringe pattern along the channel surface created from the specklegram taken for a 3-mm spaced vertical isothermal channel. The location $X_f$ where Young’s fringes completely disappear can be defined as the point of merging of the two separate boundary layers into a fully developed laminar convection channel flow. This is because the temperature gradient at the wall surface reduces to nearly zero once the flow is fully developed. Measurements showed that $X_f$ first increased with the surface-to-ambient temperature $\Delta T$ and then decreased with $\Delta T$ (Fig. 19). This phenomenon is the well-known thermal choking and can be predicted when the concept of thermal drag is introduced. The induced mass flow rate and heat transfer increase with an increasing channel wall temperature. However, once the wall temperature exceeds a critical temperature the induced mass flow rate and heat transfer decrease with increasing wall temperature [49].

The flow reversal occurring in the vertical channel flows can describe the nature of thermal choking. Figure 20 shows the numerically calculated streamlines and isotherms for the aspect ratio $L/b = 8$, where $L$ is the vertical channel length and $b$ is half the channel width [50]. The modified Rayleigh number is [51]

$$Ra' = Gr \frac{v}{\alpha} \frac{b}{L} = Ra \frac{b}{L} \frac{g\beta \cos \gamma (T_w - T_\infty) b^3}{\nu^2} \frac{v}{\alpha} \frac{b}{L}$$

![Fig. 18. Experimental determination of the length $X_f$ of the developing region by time changes in Young's fringes [48].](image)
With increasing Ra*, the flow in the channel progresses to deplete streamlines from the channel center, showing the separate boundary layer development. The more concentrated streamlines appearing near the wall suggest increased buoyancy-driven air flow. This increased convection also reduces the penetration of the conduction effect at the inlet, as seen in the isotherm distributions.

When Ra* = 500, exceeding the critical Rayleigh number of 146 (Fig. 21), the pocketlike streamlines show the formation of recirculating flows. Vena-contracta-like streamlines at the entrance appear to reduce the effective opening. This effectively narrowed opening reduces the incoming air, whereas the increased thermal driving force requires more air flow.

![Diagram showing flow and isotherms](image)

**Fig. 19.** The length $X_f$ versus the surface-to-ambient temperature difference $\Delta T = T_s - T_{\infty}$ [48].

![Streamlines and isotherms](image)

**Fig. 20.** Calculated flow streamlines and isotherms of convective air ($Pr = 0.7$) for $L/b = 8$: (a) $Ra^* = 1.0$; (b) $Ra^* = 100$; and (c) $Ra^* = 500$. The photographs indicate smoke stream visualization at $Ra^* = 1301.2$ and 5011.8, respectively [50].
When a critical point is reached, the incoming air flow through the inlet is insufficient and additional air is drawn in through the central portion of the channel exit, resulting in flow reversal. The isotherms show that the temperature in the recirculation region is much lower than it is in the convecting flow. This suggests that the convection heat transfer to the recirculating air from the wall is limited and that the presence of the flow reversal decreases the overall heat transfer.

Figure 22 shows the laser speckle data of the local Nusselt numbers for vertical channel flows of different aspect ratios of b/L, where b is the channel width, and L the channel length [52]. For opening ratios down to 0.1, where each wall acts as an independently developing separate boundary layer, the results showed good agreement with the well-established theory of a single vertical isothermal wall of Ostrach, Nu = 0.5046(Gr/4)^0.25 [43]. For b/L = 0.05, however, the data began to deviate from the theory when the local Grashof number exceeded 5 x 10⁴. The two thermal boundary layers began interfering with each other, which resulted in a decrease in the wall temperature gradients and Nusselt numbers because of thermal choking. When the local Grashof number was larger than 10⁵, as in b/L = 0.05, the merged flows prevail over uniform temperature profiles across the channel giving no measurable fringes.

Given the condition in Fig. 22, the minimum channel opening for separate layer development was estimated to be (b/L)_{min} = 0.0584 using the experimental correlation of Bar-Cohen and Rohsenow [53]. This is consistent with the experimental finding of b/L = 0.05, where the thermal boundary layers are expected to merge and interact.

![Figure 21](image1)

Fig. 21. Experimental and numerical results of the penetration length versus Ra* and numerically predicted onset Rayleigh numbers for flow reversal [50].

![Figure 22](image2)

FIG. 22. Local Nusselt number versus local Grashof number for an isothermal vertical channel flow for different aspect ratios [52].
E. UPWARD-FACING ISOTHERMAL SURFACES

The upward-facing isothermal plate (Fig. 23) has been studied in many different ways, but is still under dispute, showing the widely scattering results of heat transfer correlation coefficients [54]. The laser speckle technique has measured the heat transfer coefficients for upward isothermal plates of different aspect ratios [55, 56]. The measured local heat transfer coefficient data were integrated along the plate to evaluate the average or global heat transfer coefficient because most existing data are available only for the average values.

![Fig. 23. Thermal flow of natural convection from an upward-facing heated plate [55].](image)

Figure 24 presents the global Nusselt number data for two rectangular isothermal plates with aspect ratios of 1.75 and 15, respectively. The speckle results fall in the band between the one-fourth power-law, an experimental correlation using an electrochemical method for a square plate [57], and the one-fifth power-law prediction, based on the similarity solution for an infinite plate using a mass transfer analogy [58]. The length scale used was traditionally accepted, that is, $x = A/P$ where $A$ is the total surface area and $P$ is the perimeter along the plate edges. The band between one-fourth and one-fifth power consists of the data and predictions of aspect ratios, ranging from 1.0 to infinity, showing a gradual decrease in Nusselt number with increasing aspect ratio.

![Fig. 24. Speckle results of average Nusselt number for a horizontal isothermal plate compared with other experimental and numerical results [56]. Ra, Rayleigh; AR, aspect ratio](image)
The local Nusselt number along the perimeter is usually higher than is the average value because the cold-induced flow creates a large temperature difference with the isothermal surface temperature. The specific perimeter, total perimeter length divided by total heated surface area, increases with decreasing aspect ratio (AR), and the maximum specific perimeter is given for a square surface (AR = 1). Thus, a surface of a smaller AR gives a higher average Nusselt number because of its relatively larger specific perimeter. This is attributed to the gradual increase in the Nusselt number toward the square surface of AR equal to one, as shown in Fig. 25. It is conjectured that the Nusselt number decreases because of the decrease in the specific perimeter per unit heated area with an increase in the AR.

![Graph showing Nusselt number versus aspect ratio for Ra = 10^4](image)

Fig. 25. Average Nusselt number versus aspect ratio of a horizontal heated plate for Ra = 10^4 [55]. Ra, Rayleigh.

F. CONVERGING CHANNEL FLOWS

Natural convection heat transfer characteristics for a unique geometry of vertically converging isothermal channel walls (Fig. 26) were studied using the laser speckle technique [52, 59]. Unlike a closed triangular enclosure with no openings [66, 67], the natural convection boundary layers along the inclined heated surface induce air flow from the lower openings. Enough top openings ventilate the induced flow (Fig. 26(a)) and enable the bottom plate (the ceiling in a building) to be effectively insulated from the heated inclined surfaces (the roof). However, when the top opening is too small (Fig. 26(b)), the two boundary layers merge inside and the heat and mass transfer of the convective air can significantly decrease, because some of the induced and heated air must recirculate within the enclosure. The phenomenon can be considered analogous to the thermal choking for a parallel channel, discussed in Section D.

Figure 27 presents results of the local Nusselt number versus the Grashof number for a 30-degree inclination from the vertical. The solid line represents Ostrach’s theory [43], which accounted for the decreased gravitational effect due to the inclination. For the wide opening of \( b/L = 0.2 \) or larger, which allowed an uninterrupted boundary layer to grow along each plate, the data are well predicted by the single plate theory. As the top opening decreased, however, the thermal
boundary layers merged inside the channel and the heat transfer along the plate significantly decreased after that. The data began deviating from the single plate theory at a critical Grashof number, and the local Nusselt number remained nearly flat. With a decrease in \( b/L \), the deviation started at a smaller Grashof number because the merging of the thermal boundary layers occurred nearer the leading edge. With a decrease in the slope, the degree of data deviations from the single plate theory became larger and the deviation started at a smaller Grashof number.

Fig. 27. Local Nusselt number versus local Grashof number for the converging flow with an inclination angle \( \gamma = 30 \) degrees measured from vertical \([52]\).
Figure 28 shows the speckle data of the average Nusselt number versus the top opening ratio \( b/L \) for the range of the inclination angle \( \gamma \) measured from the vertical. When \( b/L \) was large, the average Nusselt number \( \overline{Nu_L} \) approached the analysis for an inclined single plate [43], despite the inclination angle. As \( b/L \) decreased, a slight increase in \( \overline{Nu_L} \) was observed until the gradual ridge was reached. This maximum heat transfer occurred at an optimum opening ratio \( (b/L)_{opt} \), which maximizes the so-called chimney effect. The vertically parallel channel of \( \gamma = 0 \) showed the smallest optimum opening ratio of \( (b/L)_{opt} = 0.07 \) (compared with \( (b/L)_{min} = 0.0584 \) estimated by Bar-Cohen and Rohsenow [53]). The optimum opening ratio increased with an increasing inclination angle or decreasing slope. When the top opening decreased beyond the optimum value, the average Nusselt number began decreasing and eventually became far smaller than the corresponding single plate limit because of the more prevailing thermal choking and recirculation of heated air inside.

The average Nusselt numbers and optimum top openings are summarized in Table 1. The optimum opening increases with an increase in the inclination angle or decreasing slope with an approximately linear relationship. The buoyancy driving force decreases with a decrease in the slope (increasing inclination angle), and the average Nusselt number decreases with the inclination angle for each opening.

### IV. Applications for Turbulent Flows with Density Fluctuation

Speckle photography can provide a full-field measurement of turbulence properties for a fluctuating density field [68]. For example, an axisymmetric helium jet exhausted at a high Reynolds number into the ambient air (Fig. 29) can generate a completely turbulent flow with a fluctuating density field [69]. The use of a pulsed ruby laser allowed freezing of the flow within its pulse duration, which was adjusted to be shorter than the density fluctuating time scale and reduced the blurring of speckle images. The inset photographs show Young's fringes generated at the respective locations on the specklegram. The different spacings and orientations of the fringes show that the angle of light deflection is irregularly distributed in the turbulent jet.

### TABLE I

AVERAGE NUSSELT NUMBER, \( \overline{Nu_L} = \overline{hL}/k \) for \( Pr = 0.71 \); \( Gr_L = \pi \beta \cos \gamma (T_w - T) L^3/\nu^2 \) ranged from \( 3.58 \times 10^6 \) (\( \gamma = 60 \) DEGREES) to \( 7.16 \times 10^6 \) (\( \gamma = 0 \)) [52].

<table>
<thead>
<tr>
<th>( \gamma ) (degrees)</th>
<th>0</th>
<th>15</th>
<th>30</th>
<th>45</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b/L )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.02</td>
<td>–</td>
<td>5.17</td>
<td>9.72</td>
<td>–</td>
<td>2.78</td>
</tr>
<tr>
<td>0.05</td>
<td>21.21</td>
<td>11.789</td>
<td>16.876</td>
<td>–</td>
<td>5.08</td>
</tr>
<tr>
<td>0.1</td>
<td>27.867</td>
<td>23.554</td>
<td>16.926</td>
<td>12.06</td>
<td>10.8</td>
</tr>
<tr>
<td>0.2</td>
<td>25.65</td>
<td>24.028</td>
<td>20.667</td>
<td>18.187</td>
<td>17.75</td>
</tr>
<tr>
<td>0.3</td>
<td>27.122</td>
<td>24.168</td>
<td>22.309</td>
<td>–</td>
<td>23.37</td>
</tr>
<tr>
<td>0.4</td>
<td>–</td>
<td>25.816</td>
<td>26.857</td>
<td>23.64</td>
<td>25.25</td>
</tr>
<tr>
<td>1.0</td>
<td>26.258</td>
<td>23.35</td>
<td>22.791</td>
<td>22.28</td>
<td>21.53</td>
</tr>
<tr>
<td>2.0</td>
<td>–</td>
<td>25.933</td>
<td>23.029</td>
<td>22.44</td>
<td>20.91</td>
</tr>
<tr>
<td>( \infty ) single-plate limit (Ostrach [43])</td>
<td>24.952</td>
<td>23.736</td>
<td>24.07</td>
<td>22.88</td>
<td>20.98</td>
</tr>
<tr>
<td>( (b/L)_{min} )</td>
<td>0.07</td>
<td>0.1</td>
<td>0.3</td>
<td>0.35</td>
<td>0.4</td>
</tr>
</tbody>
</table>
Section A discussed a laminar flow of inhomogeneous density field and Section B presents a measurement of the spatial correlation functions and other statistical turbulence properties for fluctuating density and temperature fields. Section C presents a single-exposure speckle photography that measures the turbulence root mean square (rms) intensities and anisotropy characteristics for turbulent flames.

A. LAMINAR AND INHOMOGENEOUS DENSITY FIELDS

Equations (3a) and (3b), with the Gladstone-Dale relationship \( n = K \cdot \rho + 1 \), give expressions for the deflection angles measured for an unknown density field at a sliced cross section \( z = z_i \) (see Fig. 7 for the coordinate system):

\[
\varepsilon_y(y, z_i) = K \int_0^L \frac{\partial \rho(x, y, z_i)}{\partial y} \, dx \\
\varepsilon_x(y, z_i) = K \int_0^L \frac{\partial \rho(x, y, z_i)}{\partial z} \, dx
\]

where \( K \) is the Gladstone-Dale constant and \( L \) is the light path length occupied by the test field in the line-of-sight \( x \)-direction. The two components of instantaneous deflection angle \( \varepsilon_y, \varepsilon_x \) represent the path integral of a density gradient.
in each direction. Tomographic inversion of Eq. (14) gives the density distribution in the x-y plane at \( z = z_f \). This inversion is repeated at different \( z \) locations to complete the reconstruction of the whole density field.

An axisymmetric field can be reconstructed from a single projected image at any arbitrary projection angle [70]. After expressing Eq. (14) for a cylindrical coordinate avstem, the use of the Abel transform gives an analytical inversion of Eq. (14) [71]:

\[
\frac{\rho(r, z)}{\rho_o} = 1 \left( \frac{1 + K\rho_o}{K\rho_o \pi} \right) \int_{y^2 - r^2}^\infty \frac{\epsilon_r(y, z)}{\sqrt{y^2 - r^2}} dy
\]

where \( \rho_o \) is the ambient density and \( r^2 = x^2 + y^2 \). The derivation of Eq. (15) assumes a paraxial ray refraction with very small deflection angles and the primary ray refraction occurring only within the x-y plane; that is, all fringes are formed perpendicular to the x-y plane. Another assumption used in Eq. (15) is a constant value for the Gladstone-Dale constant \( K \) through the whole field.

Tomographic reconstruction requires multiple projections for a nonaxisymmetric field. In Fig. 30, the nonrotating x-y coordinate is fixed on the cross-sectional plane. The \( s-t \) coordinate rotates with the projection angle \( \theta_i \), where the \( s \)-coordinate is parallel to the speckle image plane at a given projection angle \( \theta_i \) and the \( t \)-coordinate is parallel to the ray direction. The z-direction is aligned with the flow perpendicular to the \( s-t \) plane. Assuming that several projections are

![Image](image_url)
available at equal angular intervals, the field reconstruction can be achieved by a direct mathematical inversion calculated simultaneously from the projected images. This is typically possible by using Fourier convolutions (FCs) based on the Fourier slice theorem [72, 73].

For a laminar helium jet into ambient air, for example, the reconstruction equation for the relative helium density was derived as a discretized form [74]. When specklegrams are taken in \( N \) different viewing directions with equal angular intervals \( \Delta \theta \), and evaluated along the \( s \)-direction at \( M \) discrete points of equal spacing \( \Delta s \) on the imaging plane, the result is

\[
\rho^*(x', y') = \frac{\rho_{He}}{\rho_{He,0}} \cdot \frac{1}{N\pi} \sum_{m=1}^{N-1} \frac{1}{M-1} \sum_{n=1}^{M-1} e^{i(m\Delta s \cdot n\Delta \theta)} \cdot q(m - m')
\]

(16)

![Diagram](image)

Fig. 30. Coordinate system of an arbitrary cross section of test field

where \( i, j, m, m', \) and \( n \) are integers, and \( \rho_{He} \) and \( \rho_{He,0} \) are the partial density of helium in the jet and the density of pure helium, respectively. The discretized filter function [75, 76] is given as \( q(m) = 1/m \) for odd \( m \), and \( q(m) = 0 \) for even \( m \).

Figure 31 shows the tomographic reconstruction of the helium jet at a cross section 5 mm above the elliptic nozzle exit. The nozzle was mounted on a turning plate and specklegrams of up to 12 viewing angles were taken with 20 × 20 mm projected image size. Each specklegram was processed by the point-by-point analysis measuring the Young’s fringe spacings and the ray deflection angles so that the relative helium density could be determined from Eq. (16). The results show that speckle photography extends its applicability to optical tomography for measuring 3D density fields. Reconstructions with less than 12 angular samplings show artifacts of the abasing effects. For more arbitrary fields, such as the human body or brain, the FCs commonly require hundreds of equal-angled projections for acceptable reconstruction with minimal aliasing effects [77].

B. FLUCTUATING DENSITY AND TEMPERATURE FIELDS

Certain statistical properties describe turbulent flow characteristics rather than spatially and temporally resolved quantities [78]. Uberoi and Kovasznay [79] found a way to derive the statistical properties of a turbulent density field once spatial correlations of an integrated optical quantity, here the light deflection angle, are obtained. The spatial correlation function \( R_{\varepsilon_j} \) of the component \( \varepsilon_j \), for example, between the points \((x, y)\) and \((x + \xi, y + \eta)\), is expressed as
Fig. 31. Tomograms of the helium density distribution in a jet of elliptic cross section; reconstructed with (a) 4, (b) 6, (c) 8, and (d) 12 speckle projection [74].

\[ R_{xy}(\xi, \eta) = \langle \varepsilon_x(x, y) \cdot \varepsilon_y(x + \xi, y + \eta) \rangle \]  

(17)

where \( <...> \) denotes an integral and averaging operation over the whole field. The spatial correlation of \( \varepsilon_y \) in the \( x \)-direction: is calculated by \( R_{xy}(\xi, \eta = 0) \), and the spatial correlation of \( \varepsilon_x \) in the \( y \)-direction is calculated by \( R_{xy}(\xi = 0, \eta) \).

To perform the correlation integration in Eq. (17) with proper statistics satisfied, thousands of point-by-point analysis for a single specklegram are required. This requires the automatic scanning and computerized fringe analysis.

For the axisymmetric and isentropic turbulent flow field, the spatial density correlation function of the radial coordinate \( R_\rho(r) \) can be derived from the spatial correlation [80]

\[ R_\rho(r) = \frac{1}{\pi L K^2} \int_0^\infty \frac{\xi}{\sqrt{\xi^2 - r^2}} R_{xy}(\xi, \eta = 0) d\xi \]  

(18)

where \( L \) is the length of the beam path occupied by the test field, and \( K \) the Gladstone-Dale constant. The density correlation can be determined strictly from the measured deflection angles. Using the equation of state and assuming constant pressure, the correlation function for the density \( R_\rho(r) \) can be converted into the correlation function for the temperature \( R_T(r) \).
Figure 32 shows the normalized correlation function of the temperature based on Eqs. (17) and (18). The deflection angle measurement was made for an axisymmetric isotropic turbulent flow parallel to the x-direction and the temperature fluctuation was generated by electrically heated wires with an approximate 800°C surface temperature [80]. A wire mesh (M = 41 mm) was used both to heat and to generate density and temperature fluctuations. The results of the spatial correlation were compared with others using a cold-wire probe [81-83]. Good agreement was found.

The spatial correlation of the cold-wire results is referred to an equivalent radial distance $\bar{U} \cdot t$, because the point-by-point cold-wire data have been taken as a function of time under a mean velocity $\bar{U}$. This determination of the spatial correlation function from the temporally measured cold-wire data is accepted under Taylor’s hypothesis [84]. Laser speckle photography allows nonintrusive and full-field measurement of a spatial correlation without the Taylor hypothesis, because the correlation function from the speckle measurement was carried out simultaneously. Another feature of the speckle measurement is its higher data resolution. The speckle interrogation for Fig. 32 was made at 0.4-mm increments, which conforms to a nearly continuous data curve.

Fourier analysis of the spatial correlation function allows for determining the turbulence energy spectrum, and the differentiation and integration of the spatial correlation curve provides micro- and macro-length scales for temperature and density fluctuations. A study has been done to determine these turbulent statistical properties from the spatial correlation function for hot air flows generated by a combination of a turbulence grid and flat flame burner [85]. The measured properties showed good agreement with the existing theoretical models and experimental data obtained by an intrusive means such as a hot-wire probe. A shock tube experiment was also performed to study a compressible turbulent flow using laser speckle photography. An amplification of the turbulence intensity by the shock-grid interaction process was observed quantitatively [86].

C. SINGLE-EXPOSURE SPECKLE PHOTOGRAPHY

Regular double-exposed speckle photography for turbulence measurements requires a short exposure time to freeze the turbulence structure. This is usually achieved with a short-pulsed ($\ll 1 \mu$sec) laser. The single-exposure speckle photography technique [87] records speckles only with the test section present, and the exposure time is set much longer...
than the turbulent fluctuating time scale. Analysis of the smeared speckle images, which result from the beam steering by the fluctuating field during the lengthy exposure period, can provide information on the turbulence characteristics.

The interrogating laser light ray is diffracted by the speckle images recorded on a single-exposed specklegram and a diffraction halo [88] forms on the imaging screen (Fig. 33). When the turbulence fluctuation is uniformly distributed, as in isotropic turbulence, the smeared speckle images and their halo are both of circular cross section (dashed circles in Fig. 34). The higher the flow fluctuation is, the larger the diffraction halo image that is recorded because of the more significant beam steering. With anisotropic turbulence, the speckles become elongated in the preferential direction of the turbulent fluctuations. The halo then has an elliptic cross section with its major axis perpendicular to the major axis of the smeared speckle image (solid ellipses in Fig. 34).

Fig. 33. Formation of diffraction halo by illuminating a specklegram with a low-power interrogating laser.

Fig. 34. Schematic of averaged speckle form obtained by single-exposure speckle photography, and diffraction halo cross section: undisturbed or isotropic turbulence (dashed circles); and disturbed or anisotropic turbulence (solid ellipses) [871].
The geometric correlation between the speckle and halo images gives an expression for a quantity \( A \) that describes the deviation of the halo cross section from circular form, or asphericity, as [89]

\[
A = \frac{\{ (\Delta d_y)^2 \}}{\{ (\Delta d_x)^2 \}} \cong \frac{D_y}{D_x} \approx \sqrt{< \Delta T^2_y > / < \Delta T^2_x >}
\]

(19)

where \( \Delta d_x \) and \( \Delta d_y \) denote the deformation of the speckle image in its minor, or, shorter, and major, or longer, axis directions, respectively. The average \( < \ldots > \) must be taken over all speckles inside the area of the interrogating laser beam cross section. The asphericity of halo image \( A \) measures the anisotropy of the temperature fluctuation on the \( x-y \) recording plane. The rms temperature fluctuation is given as

\[
\sqrt{< \Delta T^2 > / T^2_\infty} \approx \frac{D_\infty}{D} - 1
\]

(20)

where \( D_\infty \) is the halo image size for isotropic turbulence.

Example results obtained from a turbulent flame are shown in Fig. 35 as contour maps, using the false gray levels for different quantities. At the instant when the specklegram was recorded, the right side of the flame showed a higher rms turbulence level and larger anisotropy of the temperature fluctuation then did the left side.

![Specklegram Examples](image)

Fig. 35. RMS values of temperature fluctuations in turbulent flame (a) and anisotropy of turbulence in flame (b), visualized by different gray scales and reconstructed from a single-exposure specklegram [87].

V. Applications for Flame Temperature Measurements

Speckle photography can nonintrusively measure temperature distributions for inhomogeneous gas fields, such as combusting flames, and is considered a potentially promising tool for the analyses of inhomogeneous flows of liquids or plasmas [90]. Part V discusses temperature measurements of axisymmetric and laminar gas flames. (Part VI will then present an application for a liquid jet flow). Section A discusses the tomographic reconstruction of the temperature field from the deflection angles recorded on the specklegrams. Section B and C discuss the temperature fields for axisymmetric candle flames and other gaseous flames, respectively.
A. TOMOGRAPHIC RECONSTRUCTION OF TEMPERATURE FIELD

Refractive indices of most gas species gradually decrease with an increase in temperature. All of these refractive index values are extremely close to that of air, that is, nearly unity [91]. The Gladstone-Dale relationship [7] provides an expression for the index of refraction of air at 1 atm as

\[
 n(T) = 1 + K \cdot \rho = 1 + K \cdot \frac{P}{RT} = 1 + \frac{0.07966}{T}
\]

where \( T \) is in K and the Gladstone-Dale constant \( K = 0.2257 \times 10^{-3} \text{ m}^3/\text{kg} \) at \( \lambda = 632.8 \text{ nm} \) of a helium-neon laser. At high-temperature environments, such as a combusting flame, the refractive index itself is low; however, more noticeably, the absolute magnitude of its gradient, \( dn/dT \) \((\text{Eq. (7)})\), approaches zero (Fig. 36). This means that the change of refractive index per unit temperature change dramatically decreases with an increase in temperature and reduces the sensitivity of any optical technique, including laser speckle photography, relying on the refractive index variation. Henceforth, the application of the speckle technique for flame temperature measurements inherently enlarges the measurement uncertainties compared with applications for flows at moderate temperature range, such as an application to natural convection.

The polar coordinate transformation of Eq. (3a) for an axisymmetric field gives the measured deflection angle \( \varepsilon(y) \) as an integral function of the radially distributing index of refraction \( n(r) \) as [12]

\[
 \varepsilon(y) = 2y \int_0^1 \frac{\partial}{\partial r} \left( \frac{n(r)}{n_o} \right) \frac{dr}{\sqrt{r^2 - y^2}}
\]

\( \text{Fig. 36. Refractive index of air 1 atm versus air temperature.} \)
where the y-z imaging plane lies perpendicular to the optical axis \( x \) (Fig. 7). Equation (22) represents an integral transform of the Abel type. Thus, the inverse Abel transform of Eq. (22) is given as

\[
n(r) = n_o \left[ 1 - \frac{1}{\pi} \int_r^\infty \frac{\epsilon(y)}{\sqrt{y^2 - r^2}} \, dy \right] \tag{23}
\]

which also is called Radon transform and allows the reconstruction of the refractive index field from the line-of-sight deflection angle data \( \epsilon(y) \) [70].

The reconstructed radial refractive index profile can be converted to the temperature field once the species concentration are known in the flame. When the deviation of index of refraction in the flame is small, the effect of combustion species can be neglected and the flame field can be approximated and represented by air. Applying the Gladstone-Dale relation \( n = 1 + K \, \rho \) once for a reference point denoted by a subscript \( o \) and for any arbitrary radial point, and then combining the two equations with an ideal gas assumption gives [12]

\[
n(r) = n_o \left[ \left( \frac{n(r)}{n_o} - 1 \right) \left( \frac{RT_o}{Kp_o} + 1 \right) + 1 \right]^{-1} \tag{24}
\]

where the pressure is assumed constant in the whole recorded field.

An alternative conversion is possible by using the Lorenz-Lorenz relation [92], \( A \cdot \rho = (n^2 - 1)/(n^2 + 2) \), where \( A \) and \( \rho \) are the molar refractivity and molar density, respectively. An ideal gas assumption with an index of refraction \( n = 1.0 \) gives an expression for local temperature as [93]

\[
\frac{T(r)}{T_o} = \left[ \left( \frac{n(r)}{n_o} - 1 \right) \left( \frac{3p_oA + 2RT_o}{3p_oA} \right) + 1 \right]^{-1} \tag{25}
\]

where a constant pressure \( p_o \) is assumed to prevail in the test field. An extensive tabulation of the Gladstone-Dale constant and molar refractivity as functions of light wavelength for different species is available from Gardiner et al. [94]. To complete the temperature conversion, either by Eq. (24) or Eq. (25), the temperature and index of refraction at one reference point, \( T_o \) and \( n_o \), must be given \( a \) \( p \)riori.

The use of speckle interferometry [29] can also determine the temperature field by analyzing the fringes appearing in a speckle interferogram. The analysis method is identical to that for wave front interferometric techniques such as Mach-Zehnder interferometry [95]. For an axisymmetric field with \( n \approx 1.0 \), the index of refraction can be given as the fringe data \( N_f \), which is equivalent to the number of crossing fringes [96]:

\[
n(r) = n_o \left[ 1 - \frac{\lambda}{\pi} \int_r^\infty \frac{dN_f}{dy} \frac{dy}{\sqrt{y^2 - r^2}} \right] \tag{26}
\]

Using the Lorenz-Lorenz equation and the assumption of an ideal gas, an equation for temperature conversion is

\[
\frac{T(r)}{T_o} = \left[ \frac{C}{T_o} \left( \frac{n(r)}{n_o} - 1 \right) \right]^{-1} \tag{27}
\]

where \( C \) is an optical constant and is equal to 0.083928 K for a helium-neon laser. The temperature of the ambient air is \( T_o \) where the refractive index is \( n_o \).
B. AXISYMMETRIC CANDLE FLAMES

Free candle flame in a quiescent environment generates a laminar steady flame and maintains its degree of asymmetry by 10% to 15% [97]. Because of this simplicity candle flames are a popular example, not only for laser speckle photography but also for other optical techniques and theoretical interpretations [98]. Figure 37 shows Young’s fringe patterns constructed at different axial (z) and transverse (y) locations on a specklegram taken for a diffusive candle flame [56]. A narrower fringe pattern represents a larger refraction angle resulting from a higher temperature gradient, and vice versa for fringes with wider spacing. A zero temperature gradient of symmetry condition along the centerline does not dislocate speckles in either direction and generates infinite fringe spacing. The temperature gradient becomes zero at larger r as the undisturbed ambient condition is approached.

Work was done on a candle flame simultaneously projected in four directions with the multiple speckle system, shown schematically in Fig. 38 [99]. The system used two pulsed ruby lasers having a wavelength of $\lambda = 690$ nm, a pulse energy of 0.5 J, and a pulse duration of approximately 0.5 msec. The candle flame was positioned at an approximately equal (defocusing) distance of 50 mm from all four ground glass scattering plates.

The reconstruction results of Fig. 39 were obtained from the four-angle speckle photographs by means of Radon transform in association with the Gladstone-Dale type conversion with an assumption of Eq. (24). The error associated in the mathematical reconstruction procedure for local temperatures did not exceed 10%. An error of an additional 10% to 15% was estimated to account for the combustion products of paraffin involving
Fig. 38. Scheme for obtaining specklegrams simultaneously in four directions: $S_1$, ..., $S_6$, scattering plates; $L_1$, ..., $L_6$, Lenses; $CL_1$, ..., $CL_6$, collimated lenses; $BL_1$, ..., $BL_6$, beam splitters; $M_1$, ..., $M_6$, Mirrors; $IP_1$, ..., $IP_6$, image planes [99].

Fig. 39. Results on reconstructed two-dimensional temperature distribution in a candle flame [99].
carbon dioxide and water that would alter the Gladstone-Dale constant. The isotherms show that most of the section develops an axisymmetric flame. The reconstructed temperature field oscillated in the boundary of the region surrounded by an isotherm corresponding to \( T' - T_c = 1100 \text{ K} \).

Isotherms for the whole field of view were created (Fig. 40) by repeating the temperature reconstruction process for different cross-sectional fields at different axial locations in a single specklegram. The isotherms ensure the approximate symmetry of a paraffin flame.

C. PREMIXED GASEOUS FLAMES

Figure 41 demonstrates an unstable and fluctuating Bunsen burner flame where the central interferogram gives an overall view of the fringe distortion occurring due to the flame at a particular instant of time [100]. Young’s fringes were generated at 16 different positions from a single specklegram. Quantitative analysis of such a sophisticated flame would need multiple projections of specklegrams at many different angles. The recovery of the cross-sectional field requires a rigorous reconstruction algorithm for the asymmetric field, using a regression method [101] or a mathematical inversion [75].

The viability of laser speckle photography for flame temperature measurement has been examined with the Lorenz-Lorenz conversion (Eq. (25)) for a propane gas burner [93]. Similar investigation of flame temperature measurement have been performed using speckle shearing interferometry.

Fig. 40. Temperature isolines in one of the vertical cross-sections of the axisymmetric candle flame: curves 1 to 8 correspond to temperatures of 400 to 1250 K; \( x \) and \( y \) in cm [97].

They showed close agreement with their thermocouple data [30]. Verhoeven and Farrell [96J used speckle interferometry to reconstruct radially distributing flame temperature data based on the conversion in Eq. (27).

Figure 42 shows the radial temperature distribution of a premixed propane Bunsen flame 50 mm downstream of the 8-mm diameter nozzle [71]. The temperature field was reconstructed from the index of refraction using the Gladstone-Dale based formula of Eq. (24). The solid curve shows a temperature profile calculated based on the specklegram data, with the centerline as a reference point and assuming the gas constant for air and the Gladstone-Dale constant in air. The speckle data shows fairly good consistency with the thermocouple (platinum-rhodium B-type) data,
Fig. 41. Shearing interferogram of the Bunsen burner flame (center), and patterns of Young's fringes obtained by the point-by-point analysis of the specklegram at 16 different positions in the field of view [100].

except for the region near the flame boundary where the speckle data underpredicts the ambient temperature. The thermocouple data, however, has not been attempted for any corrections of the radiation loss from the probe surface nor

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**Propane Flame Temperature Profiles**

![Graph showing temperature profiles](image)

**Fig. 42.** Comparison of temperature profile obtained from deconvolved refractive index with thermocouple-measured temperature data for the axisymmetric propane Bunsen flame [71].
for the conduction heat loss through the probe stem [102]. The probe corrections must know the emissivity of the probe surface, the time constant of the probe material, and the flow Reynolds number. The temperature correction for the heat losses could upshift the thermocouple data as much as 20% near the flame center [103].

The specific gas constant and the Gladstone-Dale constant must be estimated for the mixture of gases presented in the flame when the mass fractions of the product gases are known or measured for the flame [104]. Since it is a primary function of the wavelength, the Gladstone-Dale constant \( K \) for air is expected to be a fairly acceptable approximation for the propane flame. However, the gas constant \( R \) varies rather significantly for gas species. The dashed curve represents a temperature profile calculated by assuming a gas constant for 100% propane, while keeping the Gladstone-Dale constant for air. The discrepancy in the ambient region has been significantly diminished in this limit, whereas the midrange temperature profile shows more deviations from the thermocouple data.

The nature of the refractive index dependence on temperature increases the measurement uncertainties in applications for high-temperature flames (Eq. (21)). Additionally, some improvements can reduce the measurement uncertainties:

1. The thermocouple probe corrections would raise the thermocouple data near the center to a noticeable degree, and this increase in the reference temperature \( T_o \) would also increase the speckle temperature data approaching the flame boundary.
2. The corrections for the nonradial angular refraction (occurring up to 20% at some locations [12]) would lower the specklegram temperature data and reduce the deviations from the thermocouple data in the midrange of the flame.
3. The discrepancy also can be attributed to the asymmetry of the flame, although is expected to be less than 5%.

VI. Applications for Liquid Temperature Measurements

Water is a dramatically different optical medium compared with air in view of light ray refraction. The index of refraction is related to temperature through the experimental correlation given in Radolovic [105]:

\[
N(T) = 1.3331733 - 0.00001 \cdot (1.936 + 0.1699T)T
\]

where \( T \) is in degrees Centigrade assuming a constant pressure of 1 atm. Figure 43 shows \( n(T) \) in the range between the freezing and condensing points at 1 atm. Compared with the refractive index of air (Fig. 36), the refractive index of water not only has higher values, but the rate of its change per unit temperature \( dn/dT \) is roughly three orders of magnitude higher than its counterpart for air. Expressions for \( dn/dT \) are

\[
\frac{dn(T)}{dT} = -0.00001 \cdot (1.936 + 2 \times 0.1699T) \quad \text{for water [105]},
\]

\[
\frac{dn(T)}{dT} = - \frac{1.075 \times 10^{-6}}{(1 + 0.00368184)^2} \quad \text{for air [28]}
\]

here, for example, \( dn/dT = 1.111 \times 10^4 \) for water at 27°C, and \( dn/dT = 2.427 \times 10^7 \) for air at 300 K.

The paraxial approximation, which has been the basic assumption for the speckle analyses of air and other gases previously herein, assumes that the ray deflection angle is negligibly small due to a small \( dn/dT \). Thus the Abel conversion formula for air (Eq. (23)) is derived based on a straight ray of a negligible ray bending in the gaseous phase object. However, the paraxial assumption may not be applicable to a water flow field because of its large values of \( dn/dT \). For a strong refraction, such as that in water, Lira [106] developed an analysis for the reconstruction of a refractive index field from the deflection angle data for the case of the axisymmetric situation assuming nonnegligible ray bendings:
Fig. 43. Refractive index of water at 1 atm versus water temperature

\[
\frac{n(r)}{n_0} = \exp \left( -\frac{1}{\pi} \int_{\pi}^{\infty} \frac{\varepsilon(y)}{\sqrt{y^2 - x^2}} \, dy \right)
\]

(31)

where \( X = rn(r) \) and \( n_0 \approx 1.0 \).

Speckle photography was examined to demonstrate the viability of its full-field temperature measurement capabilities for a transparent liquid medium [107]. The experiment was carried out for an axisymmetric hot water jet discharging vertically upward from a 5.9-mm diameter tube, as shown in Fig. 44. The measured deflection angle data were converted to the radial profile of the refractive index using Eq. (31), and the temperature was recovered by solving Eq. (28). Figures 45(a) to 45(c), respectively, show the fringe spacing, deflection angle, and water temperature measured at 22 mm above the discharge point.

Fig. 44. A hot water laminar jet discharging into water at room temperature [107].
VII. Concluding Remarks

Speckle photography shows promising potential for nonintrusively studying thermal flow behaviors of both laminar and turbulent gas flows, flames, and liquid flows. The speckle technique extends its scope by employing a tomographic scheme to reconstruct the cross-sectional properties from the line-of-sight speckle data.

Laser speckle photography accurately determines the heat transfer correlation coefficients for natural convection problems. In addition, Young’s fringes generated by the point-by-point analysis of the specklegram provide full-view qualitative information on the primary direction and intensity of heat transfer. The speckle system with a pulsed laser measures the statistical turbulence properties by detecting the density and temperature fluctuations. A unique single-exposure speckle system measures turbulence rms and anisotropy using a continuous wave (CW) laser. Another

Fig. 45. (a) Fringe spacing as a function of relative coordinate y measured directly in the specklegram; (b) refraction angle Φ as a function of absolute coordinate y, measured with respect to the center of the jet; and (c) temperature profile as a function of radial coordinate y [107].
multiple specklegrams. Finally, the technique shows its extended capability in measuring liquid temperature where the refractive index variation is more pronounced by a factor of approximately one thousand times.

**Nomenclature**

- \( A \) heated surface area, or asphericity of a deflection halo image
- \( a \) object distance
- \( b \) image distance, or the channel width
- \( c \) defocusing distance
- \( D \) grain size of optical deflector or ground glass, or the diffraction halo dimension
- \( d \) distance measured from a specklegram to the fringe imaging plane, or speckle dimension imaged on specklegram
- \( \gamma \) common paper, by Kr. A. number,
- \( (\beta \cos \gamma (T_2 - T_1) b^3)^{1/2} \)
- \( \gamma^* \)
- \( g \) gravitational acceleration
- \( H \) distance measured from optical deflector to an observer
- \( h \) convection heat transfer coefficient
- \( K \) Gladstone-Dale constant (for air,
- \( K = 0.2257 \times 10^{-6} \text{m}^2/\text{kg at } \lambda = 632.8 \text{ nm} \)
- \( k \) thermal conductivity
- \( L \) test field dimension parallel to the optical axis, or the channel length measured parallel to the convective flow direction
- \( M \) number of data realization points in the x-direction on specklegram
- \( m \) magnification of the second parabolic mirror
- \( m' \) magnification of camera
- \( N \) number of viewing or projecting angles
- \( n \) refractive index of medium
- \( N_f \) fringe reading from interferogram
- \( N_u, N_{u_1} \) Nusselt number, \( h \times k \)
- \( \eta \) dimensionless distance, \( y / \delta, \) or arbitrary length scale in the y-direction with \( b \) being boundary layer thickness
- \( \theta \) fringe orientation angle measured from horizon, or dimensionless temperature, \( (T - T_2)(T_2 - T_1), \) or projection angle
- \( \lambda \) laser wavelength \( (\lambda = 632.8 \text{ nm for helium-neon laser}) \)
- \( \gamma \) kinematic viscosity
- \( \zeta \) arbitrary length scale in the \( x \)-direction

**Greek Symbols**

- \( \alpha \) ray deflection angle measured before the second parabolic mirror, or thermal diffusivity, \( k / \rho a \)
- \( \beta \) volumetric coefficient of expansion
- \( \gamma \) inclination angle measured from vertical
- \( \Delta \) amount of speckle dislocation on the recording plane
- \( \delta \) amount of speckle dislocation on the defocusing plane, or boundary layer thickness
- \( \epsilon \) ray deflection angle measured after the second parabolic mirror
- \( \xi, \xi_2 \) ray entering and exiting points of test section boundary
- \( \rho \) medium density
- \( \rho^* \) relative helium density, Eq. (16)
- \( \kappa \) length scale, \( r n(r) \)

**Subscripts**

- \( \min \) minimum
- \( o \) ambient condition
- \( \text{opt} \) optimum
- \( r \) reference point
- \( w \) wall
- \( \infty \) ambient condition away from the heated wall

**References**