Tomographic-image reconstruction using a hybrid genetic algorithm

Donald P. Lyons and Ken D. Kihm

Department of Mechanical Engineering, Texas A&M University, College Station, Texas 77843-3123

Received January 14, 1997

An improved tomographic-image reconstruction method is proposed that uses a hybrid genetic algorithm (GA) that hybridizes a conventional GA and a concurrent simplex method. For the purposes of discussion, an axisymmetric phantom density field is used with an interferometric optical projection. Tomographic-image reconstruction using the hybrid GA not only improves the convergence over the pure GA but also significantly reduces the computation time.

Given a density field $\rho(x,y)$, an algebraic or linear projection, such as an interferometric projection,[1] integrates the density field along each ray (Fig. 1). Thus the measured projection is

$$\psi_i = \psi(t_i;\theta_i) = \text{const.} \int_{s_i} \rho(x,y)\mathrm{d}s_i \,, \qquad (1)$$

where s_i and t_i denote the coordinates parallel and perpendicular to the ray, respectively; θ_i denotes the projection angle of the ith projection; and the constant is a function of incident laser wavelength. Tomography involves numerical programming for reconstruction of unknown field $\rho(x,y)$ from its measured projections ψ_i for i=1,2,...,N.

Conventional tomography algorithms belong to either of two major categories. One is a direct mathematical inversion based on the Fourier slice theorem,[2] and this usually requires many equal-angled projections for one to ensure the algorithm's mathematical stability and acceptable accuracy. The other class of algorithms use successive iterations of regression optimization and are-exemplified by the algebraic reconstruction technique[3,4] (ART). The ART requires fewer projections but is an essentially monotonic algorithm. Once the ART converges to a local peak, it tends to lock itself without searching for higher peaks. In addition, the ART, as with other commonly used techniques, works for a linear reconstruction problem in which a single set of unknown parameters is optimized. A standard example of this is using a basis function of a fixed shape with an adjustable height. Nevertheless, the ART has been the most prominent iterative reconstruction technique used for tomography because of its straightforward nature. In this Letter a more flexible unknown-field description is proposed that uses a basis function described by a multiple set of parameters, for example, its height and spread.

Application of the genetic algorithm[5,6] (GA) to tomography has shown potential for overcoming the problems with existing methods, particularly for the case of limited projections.[7] Despite its robustness and potential for improvement, the GA-based tomography requires excessive computational time.[8] A hybridized GA with a downhill Simplex method is proposed to accelerate the calculation procedure without sacrificing the advantages of the GA-based tomography.

For the purpose of this discussion, a simple axisymmetric density field is considered a reference field (Fig.2), i.e., $\rho(r) = K_a \exp(-K_b r^2) + K_c r \exp(-K_d r^2)$ ($K_a = 0.5$, $K_b = 10$, $K_c = 5$, and $K_d = 10$). The reconstructing field is assumed to be conformed by a summation of basis functions[9] f, which is defined by two parameters, such as in a Gaussian function:

$$\rho(r) \sum^{10} f(r - r_i; A_i, t_i)\,, \qquad (2)$$

where Ai is the height of the basis function located at $r = r_i$ and t_i is its shape factor.

Combining Eqs. (1) and (2) leads to a system of nonlinear equations:

$$\Psi = W(O) \qquad (3)$$

where $\psi = (\psi_1, \psi_2, \psi_3, \ldots)$ is known as the measurement vector, whose components represent the projected values; W is the nonlinear function specified depending on the type of line-of-sight projection method used; and $O = (Al, A2, A3, \ldots, t_1, t_2, t_3, \ldots)$ is the object (solution) vector whose components represent the height and spread of the basis function.

Note that the selection of a two-parameter basis function makes the inversion of Eq. (3) nonlinear, and the ART becomes unusable because of the difficulty in

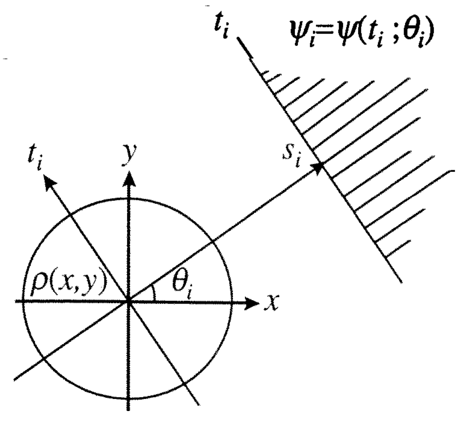


Fig. 1. Cross-section density field $\rho\ (x, y)$ and its line-ofsight projection $\Psi_I = (\theta_i, t_i)$

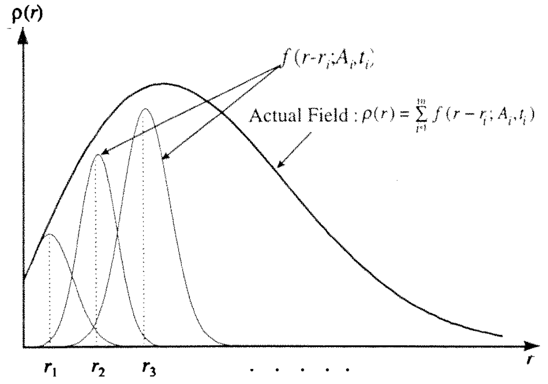


Fig. 2. Axisymmetric reference density field conformed by the summation of concentric basis functions.

properly assigning the single set of feedback information from the projected data for the two sets of unknown parameters. This conflict owing to the nonlinearity can be alleviated by the use of a nonlinear and combinatorial optimization such as a genetic algorithm that does not rely on direct feedback.

The tomographic-image reconstruction in the current context is now reduced to finding optimized amplitudes and shape factors for all basis functions, as demonstrated in Eq. (3). The evolution process of the GA can effectively optimize the finding of these unknowns (Fig. 3). Unlike conventional optimization methods, the GA starts with a randomly generated group (initial population) of solution candidates (individuals). Each individual carries randomly generated magnification and shape factors (genes) for all the basis functions. The essential feature of the GA is that the solution evolves by an internal manipulation of the genes within the population group under specified rules.[7]

The deviation of each individual's projection from the true projection of the actual field is measured by a fitness value defined as

$$F_j = \int_0^R [\psi(r) - \psi^*(r)]^2 \mathrm{d}r, \tag{4}$$

where $\psi(r)$ represents the measured projection of the unknown field and $\psi^*(r)$ represents a virtual projection of the jth individual. The term fitness is specified by the reciprocal of the fitness value. The nature of the evolution is governed by the individual fitness or survival probability to its environment, and the fitness value of the GA-based tomography is a measure of how accurately the individual solution candidate represents the actual field.

The selection operator selects individuals based on a probability associated with their fitness (biased random basis). This ensures environmental pressure, forcing survival of the fittest. Once a couple is selected, it has a prespecified chance of creating two children through exchanging mutual genes by the crossover operator. During each gene transfer, there is a prespecified chance of a gene's being altered by the mutation operator. The mutation operator is responsible for exploring new points in the search space and is the primary source of robustness.

Each population is operated on by these operators, and the next generation evolves with the same size as the current generation. The new generation is then operated on in a similar manner for evolution of further generations. This process ceases when the best individual of the present generation conforms to the actual field to a desired degree of accuracy or a specified number of generations is reached. GA's are robust and powerful methods of optimization, but they converge slowly to the optimum solution. A hybrid GA is devised to address this concern.

Figure 4 shows an illustrative example of the basic downhill simplex[10] scheme in which a population of three individuals is optimized for a close approach to the peak. Individual I_3 is the worst in terms of fitness in the first generation, and I_3 is then reflected with respect to the center of gravity of better individuals, I_1 and I_2. The newly created $I_{3'}$, happens to be the best, and I_2 becomes the worst in the next generation consisting of I_1, I_2, and $I_{3'}$. Thus I_2 is reflected with respect to the center of gravity of I_1 and $I_{3'}$ for a better individual $I_{2'}$ for the following generation. The reflection procedure is repeated until the best individual approaches a specified range from peak or the number of generations reaches the specified limit.

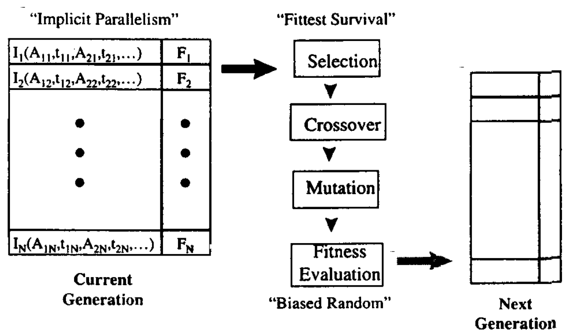
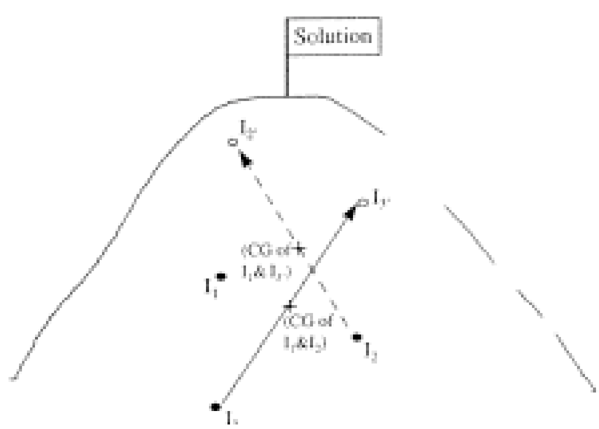


Fig. 3. Schematic illustration of the evolution of the GA for a population of solution candidate individuals containing the magnification factors conforming to the basis functions.



Fib. 4. Schematic illustration of a simple optimization

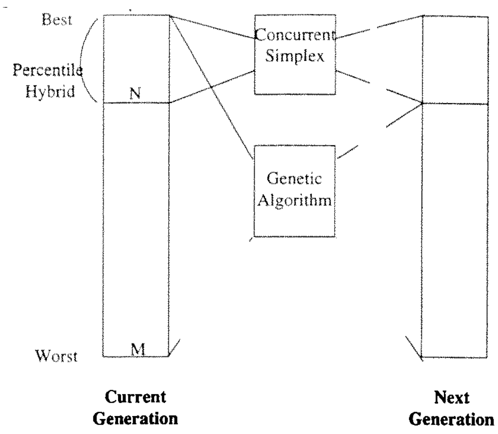


Fig. *5*. Schematic illustration of a hybrid GA with a concurrent simplex.

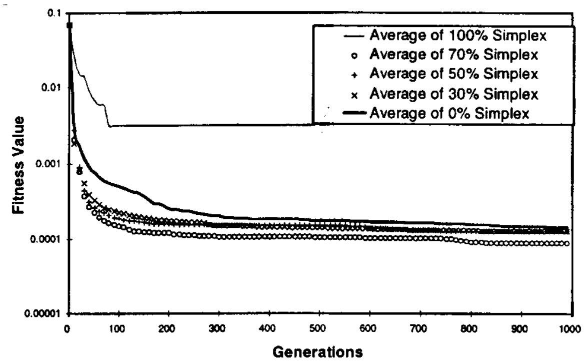


Fig. 6. Convergence of fitness values of hybrid with different portions of the simplex.

In the hybrid GA each new generation is partially created by the concurrent simplex[11,12] and partially evolved by the traditional GA operators (Fig. 5). In the concurrent simplex, multiple worst individuals, instead of a single worst individual, are selected, and they are reflected with respect to the center of gravity centroid of the remaining (better) individuals. In the traditional GA, if specific steps are not taken to ensure that the best individual in the population is maintained, the best individual may not survive to the next generation. With the concurrent simplex operator, the best *N* individuals, forming a prestigious group, create new individuals for the next generation that are guaranteed to be better than their predecessors. As a parallel development, the traditional GA evolves from the entire population (M) to fill up the remainder (M − N) of the population of the next generation based on their fitness values. The concurrent simplex exploits the continuous real-valued nature of the search space, providing, it is hoped, a good direction toward the optimum solution.

In this preliminary examination the specified chances of crossover and mutation were given as 50% and 1%, respectively. Ten concentric Gaussian basis functions were placed as shown in Fig. 2 so that they would conform to the axisymmetry reconstruction field. We carried out a series of computations to examine five different hybridization percentiles of 0% (pure GA), 30%, 50%, 70%, and 100% (pure concurrent simplex). A single initial population group was used for all the calculations to ensure that all runs for all degrees of hybridization began with the same initial condition. For each percentile hybridization, we performed ten individual calculations to average the different random number sequence of each calculation. The 100% simplex method has no random characteristics, so only one run for the given initial population was necessary.

Convergence histories of fitness values are shown in Fig. 6. As expected, the 100% simplex does not converge well, exhibiting locking after less than 100 generations. This locking presumably exhibits a local peak trap. Any level of hybridization accelerates the convergence over the pure GA case, with 70% hybridization being the best for the present case. There are several parameters that affect the performance of the hybrid GA optimization. Several of these parameters are indeed stochastic and randomly assigned during the iterative process. The performance also depends on the nature of the reconstructing field. At present, it does not seem straightforward for us to conjecture precisely as to the reason why 70% is better than the other cases.

More importantly, the results by the hybrid GA reach asymptotic value around the 150th generation, while the conventional GA does not approach an asymptotic value until nearly the 400th generation. This shows a noticeable reduction in the computational time, providing a better convergence. Although it is not presented here, research in progress has shown that similar results are observed for an asymmetric field.

References

1. C. M. Vest, *Holographic Interferometry* (Wiley, New York, *1979).*
2. A. C. Kak and M. Slaney, *Principles of Computerized Tomographic Imaging* (IEEE Press, New York, *1987),* Chap. *3.*
3. R. Gordon, IEEE Trans. Nucl. Sci. *21, 78 (1974).*
4. A. J. Decker, NASA Rep. *106330* (NASA, Washington, D.C., *1993).*
5. D. E. Goldberg, *Genetic Algorithms in Search, Optimization, and Machine Learning* (Addison-Wesley, New York, *1989),* Chaps. 1 and *2.*
6. J. Holland, *Adaptation in Natural and Artificial* Sys*tems* (U. Michigan Press, Ann Arbor, Mich., *1975).*
7. K. D. Kihm and D. P. Lyons, Opt. Lett. *21, 1327 1996).*
8. K. D. Kihm, K. Okamoto, D. Tsuru, and H. S. Ko, Exp. Fluids *22, 137 (1996).*
9. K. M. Hanson and G. W. Wecksung, Appl. Opt. *24, 4028 (1985).*
10. J. A. Nelder and ft. Mead, Comput. J. *7, 308 (1965).*
11. D. Randolph, "A hybrid of the genetic algorithm and concurrent simplex, M.S. thesis (Texas A&M University, College Station, Tex., *1994).*
12. J. Yen, J. C. Liao, D. Randolph, and B. Lee, in *Proceedings of the 11th Conference on Artificial Intelligence for Applications* (IEEE Computer Society Press, Los Almitos, Calif, *1995), pp. 277-283.*